PREDICTION OF PRECIPITABLE WATER VAPOR CONTENT WITH AN ARTIFICIAL NEURAL NETWORK BASED ON GPS DATA

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The atmosphere has been modeled for the purpose of analysis, short-term weather forecast, regional climate projections (Nelson et al., 2013), which are produced and used by the world's major research and weather forecast centers. Precipitable water vapor, which forms an essential component of the weather and climate system, is a measure of water vapor content in the atmosphere.

Precipitable water vapor is often measured at various times on Earth using different methods and instruments such as Vaisala, Micron, SLR, or DORIS. It plays a crucial role in atmospheric processes that occur over a wide range of temporal and spatial scales, from global climate to micro-meteorology. Water vapor is the most volatile of the major constituents of the atmosphere. The neutral atmosphere is a mixture of dry gases and water vapor (Bevis et al., 1994). Water vapor plays a unique role in this mixture because it is the only constituent that possesses a dipole moment contribution to its refractivity. In fact, for microwaves, its refractivity is dominated by the dipole component.

The amount of precipitable water vapor (PW) contained in the neutral atmosphere can be inferred from the propagation delay of Global Positioning System (GPS) signals passing through the troposphere. Throughout much of the troposphere, the dipole component of the refractivity is...
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Pilapanta, Christiana; Romero, Ricardoa; Porras, Luisa; Tierra, Alfonsoa.
bGrupo de Investigación Geoespacial, Universidad de Fuerzas Armadas - ESPE. Sangolqui-Ecuador. christian.pilapanta@mail.igm.gob.ec; luis.porras@mail.igm.gob.ec; ricardo.romero@mail.igm.gob.ec

ABSTRACT: The Global Positioning System (GPS) consist of a constellation of satellites that transmit of radio signals to large numbers of users engaged in navigation, time transfer, and relative positioning. These L-band radio signals are delayed by atmospheric water vapor while they travel from GPS satellites to ground GPS receivers. Water vapor plays a crucial role in atmospheric processes that act over a wide range of temporal and spatial scales, from global climate to micrometeorology. Water vapor is the most variable of the major constituents of the atmosphere. Atmospheric scientists have developed a variety of means to measure the vertical and horizontal distribution of water vapor. In this way, was studied a model for the prediction of precipitable water content from GNSS processing using a neural network. In this case was used as the initial parameters: Pressure, temperature, positions and zenith total delay. For the neural network was used one of type a radial basis function neural network (RBFNN) with three layers. Results demonstrated that the RBFNN achieved to predict precipitable water vapor with a RMSE until to 2 mm in overall GNSS network, it means, this methodology is a valuable alternative to establish a modeling for these conditions and parameters.

Keywords: Atmospheric water vapor, Global Positioning System, GPS parameters, Precipitable water content, neural network, radial basis function

1. Introduction

The atmosphere has been modeled for the purpose of analysis, short-term weather forecasts, and climate projections (Nilsson et al., 2013). Such models, which are produced and used by the world’s major research and weather forecast centers, are based on the principles of atmospheric physics which frame an analysis of the weather and climate system.

Space geodesy refers to observations that are transmitted or received by natural or artificial objects outside the lower portions of the atmosphere, i.e., in space where the density of the atmosphere is sufficiently small to allow stable satellite orbits. Space geodetic techniques like VLBI, GNSS, SLR, or DORIS are used to observe Earth rotation variations, a large part of which is caused by atmospheric effects.

The Global Positioning System (GPS) consist of a constellation of satellites that transmit of radio signals to large numbers of users engaged in navigation, time transfer, and relative positioning (Bevis et al, 1994). These L-band radio signals are delayed by atmospheric water vapor as they travel from GPS type of normalization satellites to ground GPS receivers. Water vapor plays a crucial role in atmospheric processes that act over a wide range of temporal and spatial scales, from global climate to micro-meteorology. Water vapor is the most variable of the major constituents of the atmosphere. The neutral atmosphere is a mixture of dry gases and water vapor (Bevis et al, 1992). Water vapor is unique in this mixture because it is the only constituent which possesses a dipole moment contribution to its refractivity. In fact, for microwaves, its refractivity is dominated by the dipole component.

The amount of precipitable water vapor (PW) contained in the neutral atmosphere can be inferred from the propagation delay of Global Positioning System (GPS) signals passing through the troposphere. Throughout most of the troposphere the dipole component of the refractivity is
about 20 times larger than the nondipole component. For this reason it has become common to treat the dipole component of the water vapor refractivity separately from the nondipole components of the refractivity of the water vapor and other constituents in the atmosphere. These two components are referred to as the "wet" and "hydrostatic" delays. Both delays are smallest for paths oriented along the zenith direction and increase approximately inversely with the sine of the elevation angle.

Since the observables of space geodetic typically are measurements of the travel time of the signals, the absorption is typically not important since it does not affect the propagation delay. Of course, absorption will affect the delay measurements by increasing the noise; higher attenuation will cause the signal-to-noise-ratio to be lower, and thus the accuracy of the measured delay will be worse (in the worst case the signal cannot be detected).

Meteorologists and GPS specialists working together should be able to design procedures that can be used to characterize the troposphere in more detail. Mathematical techniques have been developed to map the delay at any elevation to delay in the zenith (or vertical) direction, and the removal of the tropospheric delay by estimation has become an integral part of precise VLBI and GPS analyses (Niell, 1996; Herring, 1992; MacMillan and Ma, 1994).

The aim of this work is to establish a model for prediction of precipitable water content of the GNSS processing using a spatio-temporary-meteorological neural network. The model can provide recommendations to make decisions related with atmospheric issues.

2. Background

2.1. Path delay in the neutral atmosphere

In space geodesy normally the travel time (or difference in travel time) between a source in space (a satellite or a quasar) to a receiver on the surface of the Earth is measured. If the variations in the refractivity over the distance of one wavelength is negligible we can use the geometric optics approximation (Nilsson et al., 2013). This means that the propagation of an electromagnetic wave can be described as a ray. When calculating the propagation time of the electromagnetic wave we thus only have to consider the refractivity along the ray path. For the propagation of the signals used in space geodesy the wavelengths are a few decimeters at most, thus in the Earth’s atmosphere this approximation will normally be valid. The electric path length \( L \) (propagation time divided by the speed of light in vacuum) of a ray propagating along the path \( S \) through the atmosphere will be:

\[
L = \int_s n(s) \, ds
\]

where \( n(s) \) is the refractive index as a function of positions \( s \) along the curved ray path \( L \), and \( G \) is the straight-line geometrical path length through the atmosphere (Bevis et al, 1992). The electric path will be longer than the geometric length \( G \) of a straight line between the endpoints of the path for two reasons (see Fig. 1). Firstly, the propagation velocity is lower in the atmosphere than in vacuum. Secondly, the path \( S \) taken by the ray is, according to Fermat’s principle, the path which minimizes \( L \). The atmospheric delay, \( \Delta L \), is defined as the excess electric path length caused by the atmosphere:

\[
\Delta L = L - G = \int_s n(s) \, ds - G = \int_s \left[ n(s) - 1 \right] \, ds + \int_s \, ds - G
\]

The refractivity \( N \) (in “N-units”, mm/km, or ppm) is related to the refractive index by:

\[
N(s) = (n(s) - 1) \times 10^6
\]

Equivalently,
\[
\Delta L = \int S N(s) ds + S - G
\]

where \( S \) is the geometric length of the actual propagation path of the ray. Similarly for microwaves, \( N \) can also be divided into a hydrostatic and a non-hydrostatic (wet) part (Nilsson et al., 2013):

\[
N = N_h + N_w
\]

Then, by dividing the refractivity into hydrostatic and wet parts, we get:

\[
\Delta L = 10 \int S N_h(s) ds + 10 \int S N_w(s) ds + S - G = \Delta L_h + \Delta L_w + S - G
\]

where \( \Delta L_h \) and \( \Delta L_w \) are called the hydrostatic and wet delay, respectively. Commonly, the effect of bending, \( S - G \), is by convention considered to be part of the hydrostatic delay. In space geodesy it is common to refer the slant delays to the delays in the zenith direction. The zenith hydrostatic delay \( \Delta L^z_h \) and the zenith wet delay \( \Delta L^z_w \) are given by (Nilsson et al., 2013):

\[
\Delta L^z_h = 10 \int h_0 N_h(z) dz
\]

and,

\[
\Delta L^z_w = 10 \int h_0 N_w(z) dz
\]

where \( h_0 \) is the altitude of the site.

2.1.1. Optical Refractivity

The refractivity of the atmosphere is a function of its temperature, pressure, and water vapor pressure (Bevis et al., 1992). Smith and Weintraub (1953) suggested the relationship:

\[
N = 77.6 \left( \frac{P}{T} \right) 3.73 \times 10^5 \left( \frac{P_w}{T^2} \right)
\]

where \( P \) is the total atmospheric pressure (in millibars), \( T \) is the atmospheric temperature (in degrees Kelvin), and \( P_w \) is the partial pressure of water vapor (in millibars). This expression is considered accurate to about 0.5% under normal atmospheric conditions (Resch, 1984). In most contexts the first term in (9) is considerably larger than the second. A more accurate formula for refractivity is provided by Thayer (1974):

\[
N = k_1 \left( \frac{P}{T} \right) Z_d^{-1} + k_2 \left( \frac{P}{T} \right) Z_w^{-1} + k_3 \left( \frac{P}{T^2} \right) Z_w^{-1}
\]

where \( k_1 = (77.604 \pm 0.014) \text{ K mbar}^{-1}, k_2 = (64.79 \pm 0.08) \text{ mbar}^{-1}, k_3 = (3.776 \pm 0.004) \times 10^5 \text{ K}^2 \text{ mbar}^{-1}, P_d \) is the density of dry air (in millibars) and \( P_w \) is the density of wet air (in millibars). The variables \( Z_d \) and \( Z_w \) are compressibility factors for dry air and water vapor, respectively. These describe the deviation of the atmospheric constituents from an ideal gas. The compressibility factor for the \( i \)th constituent of air is given by

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\[ Z_i = \frac{p M_i}{\rho_i R T} \]  

(11)

where \( M_i \) is the molar mass and \( R \) is the universal gas constant. For an ideal gas we have \( Z = 1 \). Owens (1967) obtained expressions for \( Z_d \) and \( Z_w \) by a least squares fitting to thermodynamic data. Using (11) it is possible to rewrite the (10) as:

\[ N = k_1 \left( \frac{R}{M_d} \right) + k_2 \left( \frac{P}{T} \right) Z_i^{-1} + k_3 \left( \frac{P}{T^2} \right) Z_i^{-1} = N_h + N_w \]  

(12)

where \( k_i = k_i - k_i \frac{M_w}{M_d} \) and:

\[ N_h = k_1 \left( \frac{R}{M_d} \right) \]  

(13)

\[ N_w = k_2 \left( \frac{P}{T} \right) Z_i^{-1} + k_3 \left( \frac{P}{T^2} \right) Z_i^{-1} \]  

(14)

where \( N_h \) is called the hydrostatic refractivity and \( N_w \) the wet (or non-hydrostatic) refractivity. The “best average” values (Nilsson et al. 2013) of the \( k_i \), \( k_2 \) and \( k_3 \) coefficients as presented by Rüeger (2002).

2.1.2. Hydrostatic Delay

Following Davis et al. (1985), the hydrostatic delay can be determined by using the hydrostatic equation:

\[ \frac{dp}{dz} = -\rho(z)g(z) \]  

(15)

where \( g(z) \) is the gravity along the vertical coordinate \( z \), and integration of (20) yields the pressure \( p_0 \) at the height \( h_0 \):

\[ p_c = \int_{h_0}^{f} \rho(z)g(z)dz = g_{ef} \int_{h_0}^{f} \rho(z)dz \]  

(16)

Instead of the height-dependent gravity \( g(z) \), we introduce the mean effective gravity \( g_{ef} \):

\[ g_{ef} = \frac{\int_{h_0}^{f} \rho(z)g(z)dz}{\int_{h_0}^{f} \rho(z)dz} \]  

(17)

and the inversion yields the height \( h_{ef} \) which is the height of the center of mass of the atmosphere above the site and can be determined with

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\[ h_{\text{eff}} = \frac{\int_{h_0}^{h} \rho(z) dz}{\int_{h_0}^{h} \rho(z) dz} \]  

(18)

Saastamoinen (1972) used the approximation for the effective height which holds for all latitudes and all seasons

\[ h_{\text{eff}} = (0.9 h_0 + 7300 \text{m}) \div 400 \text{m} \]  

(19)

and where

\[ g_{\text{eff}} = g_m f(\theta, h_0) \]  

(20)

with the pressure \( p_0 \) at the site, it is now possible to determine the zenith hydrostatic delay:

\[ \Delta L^z_e = 10^6 \frac{k_1 R p_0}{M_d g_{\text{eff}}} \]  

(21)

with \( g_m = 9.7840 \) and

\[ f = (1 - 0.00266 \cos(2 \theta) - 0.28 \times 10^{-6} h_0) \]  

(22)

where \( \theta \) and \( h_0 \) are latitude and orthometric (or ellipsoidal) height of the station. After substitution of all values we get for the zenith hydrostatic delay in meters:

\[ \Delta L^z_e = 0.0022768 \frac{p_0}{f(\theta, h_0)} \]  

(23)

where \( p_0 \) is in hPa.

2.1.3. Wet Delay

From (8) the zenith wet delay is (Nilsson et al, 2013):

\[ \Delta L^z_w = 10^6 \left[ k_2 \left( \frac{p_w}{T^w} \right) \int_{h_0}^{\infty} \left( \frac{p_w}{T^w} \right) \right] + \int_{h_0}^{h} \left( k_3 \left( \frac{p_w}{T^w} \right) \right) dz \]  

(24)

The derivation of a model to account for the zenith wet delay \( \Delta L^z_w \) is by far more challenging than the one for the hydrostatic delay. This is due to high spatial and temporal variability and unpredictability of the amount of water vapor. Thus, the temperature and the water vapor content at the Earth surface are not representative for the air masses above. This is the reason why numerous models have been developed over the past few decades for the wet delay, while preserving Saastamoinen’s model (with slight modifications) for determining the hydrostatic delay. Saastamoinen (1972) proposes the calculation of the zenith wet delay \( \Delta L^z_w \) based on ideal gas laws using a simple relation:

\[ \Delta L^z_w = 0.0022768 \left( 1255 + 0.05 T_0 \right) \frac{p_{wd}}{T_0} \]  

(25)

where \( p_{wd} \) is the water vapor pressure and \( T_0 \) is the temperature at the surface. If no surface meteorological observation is available, we can use the simple model of the standard atmosphere.

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where $p_w$ can be calculated as a function of the relative humidity $f$

$$p_w = \frac{f}{100} \exp \left( -37.2465 + 0.213166T - 0.000256908T^2 \right)$$  (26)

2.1.4. Precipitable Water Vapor

The integrated water vapor in zenith direction can also be provided as precipitable water vapor (PWV) which corresponds to the height of the equivalent water column above the station (Nilsson et al, 2013):

$$PWV = \kappa \Delta L_v$$  (27)

with $\kappa$ defined as

$$\kappa = \frac{\Pi}{p_{w, \beta}}$$  (28)

with

$$\Pi = \frac{10^6 M_w}{k_2 + \frac{k_3}{T_m}} R$$  (29)

then combining (27), (28) and (29) we obtained:

$$PWV = \frac{10^6 M_w}{\left( k_2 + \frac{k_3}{T_m} \right) R} \frac{1}{\rho_{w, \beta}} \Delta L_v$$  (30)

The partial derivative of $\Pi$ with respect to the mean temperature yields:

$$\frac{\partial \Pi}{\partial T_m} = \frac{10^6 M_w k_3}{R \left[ k_2 + \frac{k_3}{T_m} \right] T_m^2}$$  (31)

this means that $\Pi$ is changed by about 20 $\text{kg/m}^3$. If $T_m$ of 270K is changed by 4K and assuming a zenith wet delay of 200 mm this corresponds to an error in the precipitable water vapor of about 4 mm.

2.2. Artificial Neural Networks (ANNs)

ANNs are computational units interconnected one another to calculate information from environment, allowing a fast performance in parallel way. ANNs are recognized by their high capability of generalization and approximation of functions for complex problems, it means that a reasonable response will be provided by similar problems although the environmental conditions or inputs change. As ANNs were based on biological neural networks, learning plays an important role in the implementation of algorithms to fit, approximate, associate and cluster data, these represent some applications and usefulness of ANNs.

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The properties of the computation process, for instance, fault tolerance, generalization, self-organization and make rules of learning through the experience, have been adapted in different topologies.

According with these properties is important to choose a suitable structure and topology for the learning of the ANN. The structural unit of an ANN is precisely a neuron, each neuron has a role in the routine of computation, a neuron is connected, through arithmetic operations, with synaptic weights \( w_i \) which represent the force of an input \( X_i \) towards an activation function \( f \) (Haykin, 1994). In figure 1, reader can observe how an artificial neuron works individually, using the components previously mentioned.

![Figure 1. Components of an artificial neuron](image)

From previous figure, all inputs are linked with a synaptic weight, depending on the topology of the neural network, inputs values are operated with weight in different ways. This product between inputs and weights are summed with a threshold or bias \( b \) to generate a weighted input to activation function. The activation function is preferable to be determinist, continuous and increasing, in most cases derivable in the sequence of the function. It limits the signal amplitude from previous layers of neurons to avoid saturation or inhibition in the state of activation which sends a signal to output neurons with an answer of the learning. This computation is summarized in equation 37.

\[
y = f \left( \sum x_i w_i + b \right)
\]  

(32)

ANNs are compound by layers each of them carry out a specific action. In a general description, there is input, hidden and output layer where input layer receives information of patterns, conditions or variables from environment that are being related with the problem at hand. The property of solving complex real-world problems is given by hidden layers because of non-linear activation function nature, in this point the reader can find out some activation functions in (Broomhead and Lowe, 1988), applied to different topologies and structures of ANNs. At the end of the structure of an ANN is the output layer which receives the weighted signal from hidden layer towards to

2.2.1. Radial Basis Function Neural Networks (RBFNN)

In this work was used a radial basis function neural network by its interpolation approach in a space high-dimensionally (Broomhead and Lowe, 1988). RBFNN are supervised, feed forward and fully connected networks where learning process consist in calculate weights and bias in a defined number of iterations until reaching a close response to desired output.

The common structure of a RBFNN comprise 3 layers of which hidden layer performances non-linear modeling with free parameters and it maps the input space onto a new space. Thus, output layer implements a linear combination on this new space to adjust linear parameters (weights) (Chen et al, 1991). The fundamentals of RBFNN derives from the theory of approximation, where function \( f \) is intended to look for:

\[
f(x) = \sum_{i=1}^{K} b_i \varphi \left( \| x - c_i \| \right)
\]  

(33)

where, \( \varphi \) is the radial basis function, \( c_i \) are \( K \) centers (weights) which have to be calculated and \( b_i \) is the bias vector for each hidden neuron. \( \| \cdot \| \) denotes Euclidean distance between inputs \( x \) and each centers. Radial basis activation function selected was a Gaussian function, described by:

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\[ \varphi \psi = -e^{\varphi \psi} \]  

(34)

where, \( \psi^2 \) represents squared euclidean distance \( |x - c_i|^2 \) and \( \sigma^2 \) is a constant associated with the distance of spread of weighted inputs. It is important to point out that each hidden neuron symbolize a Gaussian function with their respective centers, they send a signal to output layer with a linear output function, an approach of RBFNN is given by:

\[ y_j = \varphi_2 \left[ b_2 + \sum_{i=1}^{K} \varphi_1 \left( \frac{x - c_i}{\sigma} \right) b_1 \right] \]

(35)

Equation 39 can be interpreted as general equation of a RBFNN where \( \varphi_1 \) is radial basis function with adjustable parameters, centers \( c_i \) and multiplied by vector bias \( b_1 \). The function \( \varphi_2 \) is generally linear function with its own parameters, weights \( (w_{jk}) \) and bias \( (b_2) \), sub index \( jk \) is the dimension of weight matrix with \( K \) elements from hidden layer and \( j \) outputs. Next figure illustrates the general equation of a RBFNN.

Figure 2. General approach of threelayer RBFNN

Adjusting weights and biases is often a hard task because criteria of selection of initialization function, commonly apriori parameters are chosen randomly and close to zero, some investigations have developed some algorithms to optimize the computation of these parameters (Chen et al., 1991; Esmaeili and Mozayani, 2009). In the first case, an orthogonal least squares learning algorithm was applied to input data obtaining a systematic selection of centers and therefore fitting the RBFNN in practical signal processing. On the other hand (Esmaeili and Mozayani, 2009) proposed a particle swarm optimization (PSO) function in which spreads and centers of Gaussian functions are taken into account to establish a strong generalization ability, revealing effectiveness and efficiency in the learning process. Since nonlinear nature of hidden layer, some strategies can be associated with optimization of weights in radial basis functions (Govindaraju and Zhang, 2009). The output of the RBFNN \( (y_j) \) is assessed comparing with desired output \( (d_j) \), it might be constituted as stop criteria of training. The error of the output layer is given by:

\[ e = d_j - y_j \]

(36)

Results from (36) are feed-forwarded to RBFNN to recalculate weights and biases, this iterative process is done until reach a reasonable output, near desired output, or training ends by number of iterations. The total error after \( \tau \) iterations is:

\[ E_t = \frac{1}{2} \sum_{j=1}^{j} \epsilon_j^2 \]

(37)

where \( j \) is the number of neurons in the output layer.

The performance of training of an ANN could be measured by various statistical methods, following evaluation criteria such as coefficient of correlation (COR), root mean squared error (RMSE), mean absolute error (MAE) and index of agreement of Willmott (Ladlani et al., 2012).

3. Methodology

The strategy of GNSS data processing gave as a result values of PWV defined by meteorological

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data from global models available in a geodetic software processing, in this study it was chosen GPT model (Global Pressure and Temperature) in Gamit/Globk (Herring et al, 2015). For the purpose of covering the majority of the surface, all National GNSS Monitoring Network was processed with this strategy which is established by 35 stations.

In order to implement the RBFNN it was necessary to choose a set of patterns (inputs, outputs) to the learning process and another set to validate the parameters calculated in the earlier procedure. Input patterns were selected according certain precedents, for instance, the derivation of PWV from ZTD (Bevis et al, 1994; Bevis et al, 1992; Hagemann et al, 2003), it demonstrates an interdependence in each variable. Although the potential use of GNSS data comes given by its high temporal resolution, the corresponding time derivative of PWV highlights changes in the atmospheric water content on the different time scales (Bordi et al, 2015). Additionally, atmospheric water vapor delays are highly correlated with the elevation of the site, spatial models that take the terrain elevation into account can therefore yield better results in zenith wet delays interpolation (Xu et al, 2011).

Furthermore, (Senkal, 2015) shows a high degree of correlation among meteorological parameters and location with PW value, it suggests a cheaper and faster training with ANN, when it is compared to meteorological methods in the estimation of PW. However, PWV time series have been proved a poor correlation with precipitation, that affects the process of Precipitable Water availability in middle atmosphere (Bordi et al, 2015).

With this background, the variables selected were location (latitude, longitude and ellipsoidal height), meteorological parameters (temperature and pressure), ZTD and epoch of occurrence with the intention of predict values of PWV.

A standard established to train a ANN is the selection of training, generalization and validation data sets, the first one is intended to calculate the parameters of the learning, in this study the conditions to assign this data set was based on spatial criteria, it means all station which round the area of interest. Generalization data set allows ANN to determine the influence of smoothness of function with respect to different inputs and validation data set is not taken in account in training process with the purpose of measuring the capability of learning of the ANN.

In this study were selected only two data sets for training and test, training and generalization data were merged into the software. In figure 3 is shown the distribution of training and test data sets within of National GNSS Monitoring Network.

**Figure 3. Distribution of data set for training RBFNN**

In total, 34 stations were taken in part in the analysis which 22 station were selected for training and generalization process. These stations cover the perimeter of application of this methodology and are located in zones with lowest and highest altitudes however test data set were chosen randomly. The next step was to implement the RBFNN with normalized inputs and output with the objective to perform the learning. After of that, starts the initialization process, the goal in initializing is the optimal computation of link weights between hidden and output layer.

Firstly, the procedure selects centers \( c_r \) randomly from the input patterns and assigns them to the links between input and hidden layer. Afterward the bias \( b_1 \) is set to a value in all hidden neurons and finally the links \( (w_{jk}, b_2) \) between hidden and output layer are calculated (Bergmeir and Benitez, 2012).

The learning function was a back-propagation function which is necessary to adapt the learning rates used to the modification of centers, spread and weights that are leading to the output layers as well as of all output neurons. Additionally, to prevent an over fitting of the RBFNN, a maximum tolerated error can be set by user, ensuring a good generalization with another input data. (Zell et al, 1998).

The implementation was achieved in R software with models provided by package RSNNS (Bergmeir and Benitez, 2012; Zell et al, 1998) with parameters for Gaussian functions. The training
process was fulfilled with following framework, listed in table 1.

Table 1. Fundamental parameters/functions implemented in RBFNN to predict PWV in Ecuador

4. Results and Discussion

The first analysis carried out in this study was a comparison between selected inputs and PWV from training data that set to define some type of correlation one another, figure bellow illustrates the affinity with PWV.

Six input data sets were studied, these were latitude, longitude, ellipsoidal height, zenith, total delay, pressure and temperature, only epoch was not studied for the reason that it determines the temporal characteristic and without this variable, the model would lose this property.

Figure 4. Graphical correlation between PWV and input variables of RBFNN. Continuous lines represent PWV values and dash lines represent individual inputs. a. Graphical correlation PWV- Ellipsoidal Latitude, b. Graphical correlation PWV- Ellipsoidal Longitude, c. Graphical correlation PWV- Ellipsoidal Height, d. Graphical correlation PWV-ZTD, e. Graphical correlation PWV- Atmospheric Pressure, and f. Graphical correlation PWV- Temperature.

Figure 4a. and 4b. does not show an explicit correlation, it implies that PWV values does not depend on spatial location, however in plot 4b it is easy to identify that PWV has an increase when longitudes approximate to coast (-80°,-81°) and Amazon (-77°, -77,5°) regions, it has sense because of dense cloudiness in these zones.

On the other hand, Height, in plot 4c, reveals a inverse relation with PWV, the reason is low values of relative humidity in high lands of Ecuador. Following previous criteria, ZTD from plot 4d presents a similar behavior to PWV; as a result of that PWV values are derivable from values of ZTD which is a component of ZTD. Finally, meteorological parameters, pressure and temperature suggest a direct relation with PWV values.

To confirm these approximations, a matrix of correlation was calculated in order to prove earlier statements, the product of the matrix appears in table 2.

Table 2. Correlation between inputs of RBFNN and PWV

From last table, it is clear that latitude does not establish a degree of interaction but, longitude seems corresponding inversely PWV in zones before specified. As It mentioned before, height has a inverse correspondence with PWV, confirming the assertion of high lands presents low values of PWV, nevertheless in coast and Amazon regions PWV values are correlated with cloudiness. Values of ZTD, Pressure and Temperature reveal a high interdependence of these variables as inputs to the RBFNN.

With this background, the neural networked was trained with RSNNS package with all input patterns, given by a multiple variations one another, next figure displays the interaction of inputs.

Figure 5. Mapping of Distribution and interaction among inputs represented in a heat map.

Above figure plotted all inputs in columns (epoch, latitude, longitude, height, ZTD, pressure and temperature) in a heat map to display the way of activation of each normalized parameter. Black color represents low values which is degraded to white color that represents high values. This technique of representation allows the supervisor interpreting heterogeneity of data sets therefore the conduct of inputs.

The training was executed with parameters indicated in Table 1, the process is evidenced in figure 6a, reaching a weight squared sum error of 22.32 from a data set of 141362 patterns. The

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training curve shows a considerable decrease in first stages of learning, then it flats near limit of iterations, it demonstrates a stabilization of error further close to optimum minimum.

The outcomes of RBFNN were compared with desired output in order to establish the coefficient of determination, denoted $R^2$, in this context, it indicates how well ANN response fit with normalized real values. It obtained a $R^2$ of 0.9971 that indicates a good training.

Figure 6. The learning process of RBFNN to predict PWV with inputs related to epoch, location, altitude, ZTD and meteorological parameters. a. The training process to adjust the error given in Eq. 41. b. Training correlation, it indicates how well ANN response fit with normalized real values. c. Validation process between calculated values by RFBNN and desired values of PWV.

Later, the validation process was done with the last data set indicated in the methodology. A similar plot was made but in this case, desnormalized data was used to decide if generalization of RBFNN can be implemented to predict confident spatio-temporary-meteorological values of PWV, the $R^2$ was 0.98, it denotes that the ANN achieves the main purpose of learning. The statistical of model was based principally on coefficient of correlation (COR), root mean squared error (RMSE), mean absolute error (MAE) and index of agreement of Willmott, displayed in table 3.

Table 3. Main Statistical Parameters from Validation Process

These results can be looked at figure 7, where a double y-axis plot illustrates in black line the real values of PWV and green line the values calculated by RBFNN model. Additionally, red line represents the error in mm of each calculated value.

Figure 7. Comparison between PWV desired (Light grey line) and calculated by RBFNN (Black line) and its error (Dash line)

5. Conclusions

The model for predicting PWV based on a Radial Basis Function Neural Network was calculated with data from processing of Ecuadorean GNSS Network with a confident accuracy around 2 mm of error.

The input pattern was constituted with parameters such as epoch, position, altitude, ZTD and meteorological parameters, some of which confirmed to share a high correlation degree with PWV, for this reason it is important to take in account spatio-temporary-meteorological values for predicting PWV, the graphical relationships between inputs and PWV have ratified this assertion.

The methodology of ANN has demonstrated to be a valuable alternative to establish a model for predicting of PWV, considering the heterogeneity of input data set and low correlation with a few parameters.

References


Bordi I, Razieli T, Pereira LS, Sutera A. 2015. Ground-based GPS measurements of precipitable water vapor and

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their usefulness for hydrological applications. Water Resources Management, 29(2), 471-486.


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Figure 1. Components of an artificial neuron
185x113mm (96 x 96 DPI)
Figure 2. General approach of three-layer RBFNN
252x142mm (96 x 96 DPI)

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Figure 3. Distribution of data set for training RBFNN
982x850mm (95 x 96 DPI)
Figure 4. Graphical correlation between PWV and input variables of RBFNN. Continuous lines represent PWV values and dash lines represent individual inputs, a. Graphical correlation PWV- Ellipsoidal Latitude, b. Graphical correlation PWV- Ellipsoidal Longitude, c. Graphical correlation PWV- Ellipsoidal Height, d. Graphical correlation PWV-ZTD, e. Graphical correlation PWV- Atmospheric Pressure, and f. Graphical correlation PWV- Temperature.

396x333mm (96 x 96 DPI)

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Figure 5. Mapping of Distribution and interaction among inputs represented in a heat map.
317x283mm (96 x 96 DPI)
Figure 6. The learning process of RBFNN to predict PWV with inputs related to epoch, location, altitude, ZTD correlation, and meteorological parameters. a. The training process to adjust the error given in Eq. 41. b. Training between calculated values by RFBNN and desired values of PWV.
Figure 7. Comparison between PWV desired (Light grey line) and calculated by RBFNN (Black line) and its error (Dash line)
396x333mm (96 x 96 DPI)
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