Stationary Frame Control of Three-Leg and Four-Leg Voltage Source Inverters in Power System applications: Modelling and Simulations

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ABSTRACT

In this project, the operation and performance of the three-leg VSI control and the four-leg VSI control are studied for grid connected applications and for autonomous operation. The voltage source inverters are modelled and simulated in the stationary frames Alpha-Beta and Alpha-Beta-Gamma in order to perform the system control action employing proportional resonant compensators. The simulations are performed in PLECS that can be used standalone or integrated with MATLAB/Simulink. The three-dimensional space vector modulation technique is implemented as a C-script for the control action of the four-leg VSI. Furthermore, the independent control of each voltage phase at the point of common coupling for standalone operation using the four-leg VSI topology is implemented in a stationary frame to deal with balanced and unbalanced loads in order to improve the power quality that is sent to the AC system.
Acknowledgements

I would like to thank my parents Sergio and Mirian, my grandparents Gonzalo and Carlota, my aunt Magu and Raquelita, for all their love.

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This master course has been financially supported by Senescyt and IFTH, which recognize the importance of human capital in the development of my country Ecuador.

Finally, I would like to thank for all the blessings I have received and also for the difficulties I have to afford, which make me grow as an individual.

Dedication

To Daniel and Cristina.

To Adriana, who always used to tell me that kindness is implicit in all the people.

“Everyone is a good person”
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1. Introduction

In the last two decades, the distributed generation have become more important as an alternative source of power. Distributed generation refers to the employ of small-scale sources of power, which are generally small renewable-energy generators to produce electricity such as wind-power turbines or photovoltaic solar panels. Some advantages of the DG technology are: less environmental impact, cheap electricity and reliable power.

In order to send the electrical energy to many users, the generator units have to be integrated into the power system through a power electronic converters, which is basically a device made of semiconductor devices, which act like switches, and other passive components (Inductors and Capacitors).

Usually, different power systems have some attributes (such as the voltage level, the frequency, number of phases and the phase angle) that cannot let a direct connection to interface with each other, for that reason it is necessary to use a power electronic converter [1]. An example of this situation is a photovoltaic (PV) power station that is a subsystem that generates a DC voltage and current, which requires a power electronic inverter to interface with a utility grid that is AC system.

The pulse width modulation (PWM) voltage source inverter (VSI), which can interface a DC system with an AC system, has been increasingly used as one of the most important building blocks for renewable energy conversion systems and for others electric power systems, such as flexible AC transmission systems (FACTS) controllers and HVDC systems. So, this device is widely used in many applications that need energy conversion. It was developed due to the advances in semiconductor technology, which offers electronic switches with higher voltage and current, and also and higher switching frequency.

Nowadays, that power quality is required, the three phase voltage source inverters (VSI) are used in high power systems, because of their reduced voltage ratings for the switches, reduced harmonic spectra and improvement of control response.

A voltage source inverter (VSI) can operate basically in two modes, connected to the grid, when the frequency is imposed by the AC system, and in stand-alone mode such as in a micro-grid. In the first case, when the voltage source converter is grid connected, the control of the power send to AC system is required, it is necessary modelling the converter device as a controlled current source. On the other hand, the stand-alone mode or autonomous operation mode requires the control of the voltage, whereby the converter is consider as a controller voltage source. From literature [2] we can find many different topologies of converters to perform this tasks. In this thesis two different topologies of voltage source inverters are studied, simulated and compared: the 3-leg VSI and 4-leg VSI.
2. Power Electronic Converters, Modulation Control Techniques, and 3D-SVPWM Implementation

A voltage source inverter (VSI) is employed to convert a DC voltage to a three-phase AC voltage with the capacity of controlling the magnitude and frequency. From literature [2] we can find many different topologies of inverters to perform this task. In this chapter, the basic topologies of the 3-leg VSI and 4-leg VSI are presented with the two-dimensional and three-dimensional space vector modulation techniques.

2.1 Two Level 3-leg Voltage Source Inverter and SVM

A circuit diagram for a two-level 3-leg VSI is shown in Figure 1, as we can see the inverter is made of six switches which consist of transistors such as IGBTs or GCTs.

![Circuit Diagram of a Two-Level Three-Phase Voltage Source Inverter](image)

Figure 1. Two level three phase voltage source inverter (3-legs)

The well-known Pulse-width modulation (PWM) technique is used to control the magnitude and the frequency of the AC voltage. Among the PWM schemes that we can find in the literature, the Space Vector Modulation is the technique that will be employed in this study.

As it is shown in Figure 1, the output voltage of each leg in the VSI is going to be the positive value of the DC link voltage or the negative value of the DC voltage, which means that each leg has two possible switching states, therefore the three-leg voltage source inverter has eight possible switching states combinations, which are shown in Table 1. Furthermore, each switching state of the 3 legs VSI can be represented by a space vector in the stationary frame $a$-$b$-$c$, where $\vec{V}_0$ and $\vec{V}_7$ are called zero vectors (states) while $\vec{V}_1$ to $\vec{V}_6$ are called active vectors (states).

<table>
<thead>
<tr>
<th>$\vec{V}_0$</th>
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Table 1: Space Vectors and Switching States in $a$-$b$-$c$ stationary frame (3leg-VSI)
Considering that the inverter is part of a balanced system:

\[ V_a + V_b + V_c = 0 \]  

(1)

Then, as mentioned in [3], one phase voltage is redundant, therefore given any two phase voltage values, it is possible to calculate the third one. In other words, the system has one voltage that is dependent on the others two. Therefore it is possible to reduce the system with three dependent variables into a system with two independent variables, for which the \( \alpha-\beta \) transformation is used. From equation (2), the values of voltages and currents in the stationary \( a-b-c \) frame are converted to \( \alpha-\beta \) orthogonal frame [4].

\[
\begin{bmatrix}
    u_{\alpha} \\
    u_{\beta}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
    1 & -1/2 & -1/2 \\
    0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix} \begin{bmatrix}
    u_a \\
    u_b \\
    u_c
\end{bmatrix}
\]  

(2)

The space vectors from Table 1 are mapped into the \( \alpha-\beta \) space vector, using the equation (2), which are described in Table 2. Since \( \alpha \)-axis and \( \beta \)-axis are orthogonal, stationary frame it is possible to appreciate better the relationship between the space vectors and switching states, the six active vectors \( \vec{V}_1 \) to \( \vec{V}_6 \) form a regular hexagon as it is shown in Figure 2, the zero vectors remain in the centre point.

<table>
<thead>
<tr>
<th>( \vec{V}_0 )</th>
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<td>npp</td>
<td>nnp</td>
<td>pnp</td>
<td>ppp</td>
</tr>
<tr>
<td>( V_{\alpha} )</td>
<td>0</td>
<td>( \frac{2}{3} V_{DC} )</td>
<td>( \frac{1}{3} V_{DC} )</td>
<td>( -\frac{1}{3} V_{DC} )</td>
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<td>( \frac{1}{3} V_{DC} )</td>
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</tr>
<tr>
<td>( V_{\beta} )</td>
<td>0</td>
<td>0</td>
<td>( \frac{1}{\sqrt{3}} V_{DC} )</td>
<td>( \frac{1}{\sqrt{3}} V_{DC} )</td>
<td>0</td>
<td>( -\frac{1}{\sqrt{3}} V_{DC} )</td>
<td>( -\frac{1}{\sqrt{3}} V_{DC} )</td>
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</table>

Table 2: Space Vectors and Switching Combinations in \( \alpha-\beta \) stationary frame (3leg-VSI)

![Diagram](image)

Figure 2. Space Vectors of 3-Leg Two Level VSI in the \( \alpha-\beta \) stationary frame
In order to calculate the time that each switching vector is active the first step is to find the region where the vector demand voltage is located. From [3], we know that if the switching time $T_s$ is small enough, then the vector demand can be considered constant in that period of time, then we can establish that the space vector demand is a linear combination of the switching vectors adjacent to it and a zero vector as it shown in the following equations:

$$T_s \overrightarrow{V_{dem}} = T_0 \overrightarrow{V_Z} + T_1 \overrightarrow{V_I} + T_2 \overrightarrow{V_{II}}$$  \hspace{1cm} (3)

$$T_s = T_0 + T_1 + T_2$$  \hspace{1cm} (4)

$$\overrightarrow{V_{dem}} = \delta_0 \overrightarrow{V_Z} + \delta_1 \overrightarrow{V_I} + \delta_2 \overrightarrow{V_{II}}$$  \hspace{1cm} (5)

Where $T_i$ is the time that the vector $\overrightarrow{V_i}$ is active, $\delta_i = \frac{T_i}{T_s}$ and $\delta_2 = \frac{T_2}{T_s}$ are the duty cycles.

Once the space vectors and the duty ratios are known, it is important to sequence them, in other words, it is necessary to choose which vector will be activated first, second and so on. From [5] we know that any sequence of the switching space vectors is valid since the average vector is always the same but each sequence influence on the power losses and the harmonic content in a different manner, so it is important to choose the best one according to the case. Figure 3 shows an example of a sequence that is called the symmetrical aligned sequence.

**Figure 3. Symmetrical Aligned Sequence**

2.2 **Two Level 4-leg Voltage Source Inverter and 3D-Space Vector Modulation**

Figure 4 shows the circuit diagram of a two-level 4-leg VSI, which have eight switches, two per leg. Each switch could be an IGBT or GCT.
Figure 4. Two level three phase voltage source inverter (4-legs)

As it is shown in Figure 4, the output voltage of each leg is going to be the positive value of the DC link voltage or the negative value of the DC voltage, which means that each leg has two possible switching states, so the four-leg VSI has four times two possible combinations which are a total of sixteen possible switching states combinations, which are shown in

Table 3. Furthermore, each switching state of the 4 legs VSI can be represented by a space vector in the stationary frame $a$-$b$-$c$.

<table>
<thead>
<tr>
<th>$\vec{V}_{0}$</th>
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<td>$V_{DC}$</td>
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<tr>
<td>$V_{af}$</td>
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<td>$V_{DC}$</td>
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<td>0</td>
<td>$V_{DC}$</td>
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<tr>
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<td>$V_{DC}$</td>
<td>0</td>
<td>$-V_{DC}$</td>
<td>$V_{DC}$</td>
<td>0</td>
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</table>

As it was mention in section 2.1, in a balanced system and according to equation (1), only two variables are independent for which the $\alpha$-$\beta$ transformation is used. When a 4 legs VSI is being used this equation is no longer valid since the system has three independent variables. Each space vector in the stationary frame $a$-$b$-$c$ can be transformed to the three dimensional orthogonal vector space $\alpha$-$\beta$-$\gamma$ through equation (6) which is shown below [4]:
\[
\begin{bmatrix}
    u_a \\
    u_b \\
    u_c
\end{bmatrix} = \frac{2}{3}
\begin{bmatrix}
    1 & -\frac{1}{2} & -\frac{1}{2} \\
    0 & \sqrt{3} & -\sqrt{3} \\
    1 & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
    u_a \\
    u_b \\
    u_c
\end{bmatrix}
\] (6)

From Table 4 it is possible to appreciate the space vectors (switching states) in the \( \alpha-\beta-\gamma \) stationary frame once the transformation (6) is applied to the space vectors in Table 3.

Table 4: Space Vectors and Switching Combinations in \( \alpha-\beta-\gamma \) stationary frame

<table>
<thead>
<tr>
<th>( \vec{V}_0 )</th>
<th>( \vec{V}_1 )</th>
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<tr>
<td>( V_\alpha )</td>
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<td>(-\frac{1}{3} ) ( V_{DC} )</td>
<td>(-\frac{1}{3} ) ( V_{DC} )</td>
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</tr>
</thead>
<tbody>
<tr>
<td>pnnn</td>
<td>pnnp</td>
<td>npnn</td>
<td>nppp</td>
<td>npnn</td>
<td>npnn</td>
<td>npnn</td>
<td>npnn</td>
</tr>
<tr>
<td>( V_\alpha )</td>
<td>( \frac{2}{3} ) ( V_{DC} )</td>
<td>( \frac{2}{3} ) ( V_{DC} )</td>
<td>( \frac{1}{2} ) ( V_{DC} )</td>
<td>( \frac{1}{3} ) ( V_{DC} )</td>
<td>( \frac{1}{3} ) ( V_{DC} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( V_\beta )</td>
<td>0</td>
<td>0</td>
<td>(-\frac{1}{2\sqrt{3}} ) ( V_{DC} )</td>
<td>(-\frac{1}{2\sqrt{3}} ) ( V_{DC} )</td>
<td>( \frac{1}{2\sqrt{3}} ) ( V_{DC} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( V_\gamma )</td>
<td>( \frac{1}{3} ) ( V_{DC} )</td>
<td>(-\frac{2}{3} ) ( V_{DC} )</td>
<td>( \frac{2}{3} ) ( V_{DC} )</td>
<td>(-\frac{1}{3} ) ( V_{DC} )</td>
<td>( \frac{1}{3} ) ( V_{DC} )</td>
<td>( \frac{1}{3} ) ( V_{DC} )</td>
<td>0</td>
</tr>
</tbody>
</table>

The vectors \( \vec{V}_1 \) to \( \vec{V}_{14} \) represent the active vectors which are non-zero switching states while \( \vec{V}_0 \) and \( \vec{V}_{15} \) are the zero vectors (states) since they produce zero voltage output. Figure 5 shows the 14 switching space active vectors in the three dimensional \( \alpha-\beta-\gamma \) frame. As it was mention in [5] these vectors describe a shape of 24-faced polyhedron. The two zero space vector corresponding to the switching states \( nnnn \) and \( pppp \) are located in the centre point.

Each of the space vector of a 3-legs VSI splits into two switching vectors, depending on switch position of the neutral leg, as shown in Figure 5. The prism/region are enumerated from I to VI.
Figure 5. Switching Space Vectors in $\alpha$-$\beta$-$\gamma$ frame (a) 3D-View (b) Top View
The vector demand voltage can be located in any of the 6 prisms shown in Figure 5, as we can from Figure 7 and Figure 8, each prism is composed for 4 tetrahedrons, so in order to calculate the time that each switching vector is active the first step is identifying which tetrahedron the vector demand voltage is located in order to know the switching vectors that are going to be active. If the switching time $T_s$ is small enough, then the vector demand can be considered constant in that period of time, so we can establish that the space vector demand a linear combination of the switching vectors that conform the tetrahedron, therefore.

$$T_s \overrightarrow{V_{dem}} = T_1 \overrightarrow{V_i} + T_2 \overrightarrow{V_{II}} + T_3 \overrightarrow{V_{III}}$$  \hspace{1cm} (7)$$

$$\overrightarrow{V_{dem}} = d_1 \overrightarrow{V_i} + d_2 \overrightarrow{V_{II}} + d_3 \overrightarrow{V_{III}}$$  \hspace{1cm} (8)$$

Where $T_i$ is the time that the vector $\overrightarrow{V_i}$ is active, $\delta_1 = \frac{T_1}{T_s}$ and $\delta_2 = \frac{T_2}{T_s}$ are the duty cycles.

Using matrix notation we have:

$$\begin{bmatrix} V_{ref\alpha} \\ V_{ref\beta} \\ V_{ref\gamma} \end{bmatrix} = d_1 \begin{bmatrix} V_{I\alpha} \\ V_{I\beta} \\ V_{I\gamma} \end{bmatrix} + d_2 \begin{bmatrix} V_{II\alpha} \\ V_{II\beta} \\ V_{II\gamma} \end{bmatrix} + d_3 \begin{bmatrix} V_{III\alpha} \\ V_{III\beta} \\ V_{III\gamma} \end{bmatrix}$$  \hspace{1cm} (9)$$

Then:

$$\begin{bmatrix} V_{ref\alpha} \\ V_{ref\beta} \\ V_{ref\gamma} \end{bmatrix} = \begin{bmatrix} V_{I\alpha} & V_{II\alpha} & V_{III\alpha} \\ V_{I\beta} & V_{II\beta} & V_{III\beta} \\ V_{I\gamma} & V_{II\gamma} & V_{III\gamma} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$  \hspace{1cm} (10)$$

So, the duty cycles are calculated using the equation below:

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \frac{1}{V_{DC}} \begin{bmatrix} V_{I\alpha} & V_{II\alpha} & V_{III\alpha} \\ V_{I\beta} & V_{II\beta} & V_{III\beta} \\ V_{I\gamma} & V_{II\gamma} & V_{III\gamma} \end{bmatrix}^{-1} \begin{bmatrix} V_{ref\alpha} \\ V_{ref\beta} \\ V_{ref\gamma} \end{bmatrix}$$  \hspace{1cm} (11)$$

Once the space vectors and the duty ratios are known, we have to generate the PWM signal that controls the 4-legs VSI, so it is necessary to choose a sequence as in SVM case.

### 2.3 Implementation of 3D-SVPWM Algorithm - PLECS C-Script

The software used to simulate the four-leg VSI for this study is PLECS, which allows us to implement the 3D SVPWM method described in section 2.2 in a C-script, which requires the vector voltage demand components in $\alpha-\beta-\gamma$ coordinates and DC link voltage value as the inputs. This code is capable of the selection of switching vectors or switching states required to reach the voltage demand, then it calculate the projection of the reference vector onto selected switching vectors, in other words, this action means to obtain the duty ratios, in order
to perform this task a look-up table with 24 matrices is setup in the code, finally it sequence the switching vectors.

**Figure 6. Logic of the C-script Code**

The selection of the space vectors adjacent to the demand vector is equivalent to know the tetrahedron where the demand vector is located in a specific period of time. Since there are 24 tetrahedrons, the first step is to identify the prism or region. As it shown in Figure 5 (b) there are six prisms or regions likewise in the conventional SVM. The criteria that is used to determine the prism consists of calculating the angle of the demand vector projection in the $\alpha$-$\beta$ plane. In order to do this calculation, the vectors components of the demand vector $V_{\text{dem} \alpha}$ and $V_{\text{dem} \beta}$ are used in equation (12), which is the same criteria used in the two dimensional space vector modulation.

$$\theta = \arctan\left(\frac{V_{\text{dem} \beta}}{V_{\text{dem} \alpha}}\right)$$

(12)

**Table 5: Angle Criteria for Prism selection**

<table>
<thead>
<tr>
<th>Prism/Sector</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$0 \leq \theta &lt; \pi/3$</td>
</tr>
<tr>
<td>II</td>
<td>$\pi/3 \leq \theta &lt; 2\pi/3$</td>
</tr>
<tr>
<td>III</td>
<td>$2\pi/3 \leq \theta \leq \pi$</td>
</tr>
<tr>
<td>IV</td>
<td>$\pi \leq \theta &lt; 4\pi/3$</td>
</tr>
<tr>
<td>V</td>
<td>$4\pi/3 \leq \theta &lt; 5\pi/3$</td>
</tr>
<tr>
<td></td>
<td>$5\pi/3 \leq \theta &lt; 2\pi$</td>
</tr>
</tbody>
</table>

Each prism/region content four tetrahedrons as we can see in Figure 7 and Figure 8, which show the tetrahedrons of Prism I and Prism II respectively, so once the region is identified, we have to select one of these four options. From [5] we know that a simple technique to select
the correct tetrahedron is based on the signs of the voltage vector demand in a-b-c reference frame. So, the four tetrahedron in each prism corresponds vectors with zero positive sign, one positive sign, two positive signs and three positive signs respectively as it explained in Table 6. Therefore a simple code is necessary to find the correct tetrahedron where the voltage vector demand is located depending on the number of positive signs in a-b-c reference frame. For this implementation the tetrahedron number 1 of each prism is related to the vectors with one positive sign, the tetrahedron number 2 corresponds vectors with two positive signs, the tetrahedron number 3 to three positive signs, and finally the tetrahedron four is related to vectors with non-positive signs as it is explained in Table 6. Figure 7 and Figure 8 shows the tetrahedrons numbered according to this criteria.

Table 6: Tetrahedron Selection Criteria according to Polarity in a-b-c reference frame

<table>
<thead>
<tr>
<th>Prism/Sector</th>
<th>Tetrahedron</th>
<th>Vectors</th>
<th>SIGN</th>
<th>Vaf</th>
<th>Vbf</th>
<th>Vcf</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>V8,V9,V13</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>V8,V12,V13</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>V8,V12,V14</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>V1,V9,V13</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>1</td>
<td>V4,V5,V13</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>V4,V12,V13</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>V4,V12,V14</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>V1,V4,V13</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>1</td>
<td>V4,V5,V7</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>V4,V6,V7</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>V4,V6,V14</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>V1,V5,V7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>1</td>
<td>V2,V3,V7</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>V2,V6,V7</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>V2,V6,V14</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>V1,V3,V7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>1</td>
<td>V2,V3,V11</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>V2,V10,V11</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>V2,V10,V14</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>V1,V3,V11</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>1</td>
<td>V8,V9,V11</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>V8,V10,V11</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>V8,V10,V14</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>V1,V9,V11</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>
Figure 7. Prism I with tetrahedrons (odd-case)

Figure 8. Prism II with tetrahedrons (even-case)
From the criteria described above the prims number and the tetrahedrons number are calculated. It means we know the active vectors that define the tetrahedrons and compose the vector demand. The next step is computing the duty cycles through equation (11) from section 2.2. This formula employs the voltage demand vector in alpha-beta-gamma frame and the components of the active vectors that define the tetrahedron in order to build the transformation matrix. Instead of setting up the active vectors in the code memory to compute a matrix inversion as in equation (11) each time we run the code, a better solution is to build the 24-matrices that characterize each tetrahedron and set up the look-up table as shown in Table 7. So, through a fast operation (linear multiplication) in the C-scrip we have the duty cycles.

The sequence that is used for this algorithm implementation in the C-scrip is the symmetrical aligned sequence shown in Figure 10, which consist in activating first the zero switching vector state (nnnn) during a quarter of its calculated time, then the first, the second and third switching vector states are activated in that order during the half of them calculated time, after this the zero switching vector state (pppp) is activated and finally all the switching states are activated in the inverse order to complete the total switching time.

```c
if (Va>= 0){
a=1;}
else{
a=0;}

if (Vb>= 0){
b=1;}
else{
b=0;}

if (Vc>= 0){
c=1;}
else{
c=0;}

TETRA=(a+b+c+3)%4+1;
```

Figure 9. Tetrahedron detection code
Table 7: Matrix selection for Duty Cycle computation - 4 leg VSI (Look up Table Method)

<table>
<thead>
<tr>
<th>Prism</th>
<th>Tetrahedron 1</th>
<th>Tetrahedron 2</th>
<th>Tetrahedron 3</th>
<th>Tetrahedron 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>[ \begin{bmatrix} 1 &amp; 0 &amp; 1 \ \frac{1}{2} &amp; -\sqrt{3} &amp; 0 \ 0 &amp; \sqrt{3} &amp; 0 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} \frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 0 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} \frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 0 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} \frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 0 \end{bmatrix} ]</td>
</tr>
<tr>
<td>II</td>
<td>[ \begin{bmatrix} 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ \frac{1}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ -\frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ \frac{1}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ \frac{1}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ \frac{1}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
</tr>
<tr>
<td>III</td>
<td>[ \begin{bmatrix} \frac{1}{2} &amp; \frac{\sqrt{3}}{2} &amp; 0 \ \frac{1}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ -\frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
</tr>
<tr>
<td>IV</td>
<td>[ \begin{bmatrix} 1 &amp; 0 &amp; 1 \ \frac{1}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \sqrt{3} &amp; 0 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} \frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} \frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} \frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
</tr>
<tr>
<td>V</td>
<td>[ \begin{bmatrix} 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ \frac{1}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \sqrt{3} &amp; 0 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} \frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} \frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} \frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ -1 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
</tr>
<tr>
<td>VI</td>
<td>[ \begin{bmatrix} \frac{1}{2} &amp; \frac{\sqrt{3}}{2} &amp; 0 \ \frac{1}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \sqrt{3} &amp; 0 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} \frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 1 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \sqrt{3} &amp; 0 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} \frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 1 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \sqrt{3} &amp; 0 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} \frac{3}{2} &amp; -\frac{\sqrt{3}}{2} &amp; 0 \ 1 &amp; \frac{\sqrt{3}}{2} &amp; 0 \ 0 &amp; \sqrt{3} &amp; 0 \end{bmatrix} ]</td>
</tr>
</tbody>
</table>
Simulation 1

The first simulation of the 4-leg VSI using the 3DPWM technique that was implemented in the software PLECS, considers a three phase balanced voltage, so the voltage demand is:

\[ V_{a\text{dem}} = 300 \sin(2\pi ft) [V] \]
\[ V_{b\text{dem}} = 300 \sin(2\pi ft - \frac{2\pi}{3}) [V] \]
\[ V_{c\text{dem}} = 300 \sin(2\pi ft + \frac{2\pi}{3}) [V] \]

where the frequency is \( f = 60 [Hz] \). The DC link voltage used for this simulation is \( V_{dc} = 600 [V] \), and the switching frequency is \( f_s = 10 [kHz] \).

Figure 11 shows voltage demand and the probe signals of this simulation, which are prism number, the tetrahedron number, and the duty cycles. Since the three-phase voltage is balanced, the voltage vector demand is confined to be in the Alpha-Beta plane, in other words, the gamma component of the voltage vector demand is zero, therefore the space vector demand crosses all sectors but activate just two tetrahedrons in each sector (tetrahedron 1 and 2), spending the same time in each one. Therefore, under balanced conditions, the signals of Figure 11 are completely symmetric.

From Figure 12 we can see the voltages at the terminals of the 4-legs VSI which is a PWM signal, while Figure 13 shows the output voltage at the end of the L-filter. As we can see the voltage follow the voltage demand signal.
Figure 11. Voltage Demand and Probe Signals – Balanced Conditions
Figure 12 Four-leg VSI AC Terminal Voltages - Balanced Conditions: (a) $V_a$ phase-neutral voltage, (b) $V_b$ phase-neutral voltage, (c) $V_c$ phase-neutral voltage,
• Simulation 2

This simulation of the 4-legs VSI using the 3DPWM technique consider the following unbalanced voltage demand:

\[ V_{a\,dem} = 0[V] \]
\[ V_{b\,dem} = 200 \sin(2\pi ft - \frac{\pi}{2})[V] \]
\[ V_{c\,dem} = 300 \sin(2\pi ft + \frac{2\pi}{3})[V] \]

where the frequency is \( f = 60[\text{Hz}] \). The DC link voltage used for this simulation is \( V_{DC} = 600[V] \), and the switching frequency is \( f_s = 10[\text{kHz}] \).
Figure 15. Voltage Demand and Probe Signals – Unbalanced Conditions-S2
Figure 15 shows voltage demand and the probe signals (prism number, the tetrahedron number, and the duty cycles). The three phase voltage is unbalanced, as it was expected the probe signals are not symmetrical, the voltage vector demand goes through all four regions, and crosses the tetrahedron number, from 1 to 3, but in this case, the vector voltage never reaches the lower tetrahedron 4 in any region. The trajectory of the vector voltage can be seen in Figure 19.

Figure 16 shows the output voltage at the end of the L-filter, which follows the voltage demand signal as expected.

![Figure 16. Output Voltages – 4-legs VSI - Unbalanced conditions-S2](image)

**Simulation 3**

The third simulation of the 4-legs VSI using the 3DPWM technique consider the following unbalanced voltage demand:

\[
V_{a\, \text{dem}} = 300 \sin(2\pi ft)[V]
\]

\[
V_{b\, \text{dem}} = 200 \sin(2\pi ft - \frac{\pi}{4})[V]
\]

\[
V_{c\, \text{dem}} = 100 \sin(2\pi ft + \frac{2\pi}{3})[V]
\]

where the frequency is \( f = 60[Hz] \). The DC link voltage used for this simulation is \( V_{DC} = 600[V] \), and the switching frequency is \( f_s = 10[kHz] \).
Figure 17. Voltage Demand and Probe Signals – Unbalanced Conditions-S3
Figure 17 shows voltage demand and the probe signals (prism number, the tetrahedron number, and the duty cycles). In this simulation, the three phase voltage is unbalanced, as it was expected the probe signals are not symmetrical, the voltage vector demand go through all four regions, but unlike simulation 2, this time the vector voltage reach the tetrahedron number 4 in two regions. The trajectory of the vector voltage can be seen in Figure 19. From Figure 18 we can see the output voltage at the end of the L-filter.

Figure 18. Output Voltages – 4-Leg VSI - Unbalanced conditions-S3

Figure 19. Voltage Trajectory-Simulations 1, 2, and 3
3. Compensators

3.1 P+Resonant Compensators

In order to control the VSI, many kinds of compensators could be employed. If the system is modelled in the rotary dq frame, proportional integral (PI) compensators have been usually used. This kind of controller has been tested in [6]. Since synchronous frame PI regulators operate on DC quantities, they can be designed easily to eliminate steady-state errors.

Some ideas have been implemented to enhance PI controller performance including the addition of a grid voltage feedforward path [1], multiple-state feedback, etc. As it was mentioned in [6] these modifications can expand the PI controller bandwidth but the main disadvantage is they push the systems towards their stability limits. Another problem, mentioned in [6], about the modified PI controllers is the possibility of distorting the line current caused by background harmonics introduced along the feedforward path if the grid voltage is distorted. Therefore, if we want to achieve good results using the PI controllers with voltage feedforward in the synchronous frame, it usually requires multiple modifications, which can be difficult to implement using a low-cost digital processor.

The proportional resonant compensator is a good alternative in the stationary frame control, getting the same transient and steady-state performance as the PI with feedforward path regulator in the synchronous dq frame.

The PR compensator transfer function is described in Equation (13), as it was mention in [6] the basic functionality of the PR controller is to introduce an infinite gain at a selected resonant frequency $\omega_0$ in order to eliminate steady state error at that specific frequency, as we can appreciate in Figure 20. This controller works similar to an integrator whose infinite DC gain forces the DC steady-state error to zero, the resonant portion of the PR controller can be viewed as a generalized AC integrator, with the advantage of tuning the resonant frequency.

$$G_c = K_p + \frac{2K_r s}{s^2 + \omega_0^2}$$  \hspace{1cm} (13)

A non-ideal PR controller described in Equation (14) can be used to avoid stability problems due to the infinite gain of the ideal PR compensator. So the gain at the resonant frequency $\omega_0$ is now finite, but it is still high enough to follow the reference signal with practically zero steady-state error.

$$G_c = K_p + \frac{2K_r \omega_0 s}{s^2 + 2\omega_r s + \omega_0^2}$$  \hspace{1cm} (14)
Figure 20. Ideal P+R controller Bode Plot (Kp=1, Ki=20, $\omega=100\pi$ rad/s)

Figure 21. Non-ideal P+R controller Bode Plot (Kp=1, Ki=20, $\omega=100\pi$ rad/s, $\omega_c=100\pi$ rad/s)
4. Three-leg voltage source inverter, modelling, and control

4.1 Current Control Design in Alpha-Beta Frame

The mathematical model of the two level 3 legs voltage source inverter considered for the current control design is based on the schematic diagram of the circuit shown in Figure 22, which is described by the following equations, it is important to mention that for this model is assumed that DC link is connected to an ideal DC voltage source:

\[ V_{ta} - V_{Sa} = L \frac{di_a}{dt} + Ri_a \]  \hspace{1cm} (15)

\[ V_{tb} - V_{Sb} = L \frac{di_b}{dt} + Ri_b \]  \hspace{1cm} (16)

\[ V_{tc} - V_{Sc} = L \frac{di_c}{dt} + Ri_c \]  \hspace{1cm} (17)

Where \( V_{ta}, V_{tb} \) and \( V_{tc} \) are the terminals voltages of the two level 3-legs voltage source inverter, \( V_{Sa}, V_{Sb} \) and \( V_{Sc} \) are the voltages at the point of common coupling (PCC), these voltages are considered known values for the current control design, for instance, it can be assumed that these voltages are imposed by the grid (voltage frequency and magnitude), but also they can be considered voltages at which the load is connected, but they are measured by a transducer. It is important to mention that dynamic of the AC system from Figure 22 is not being...
considered since the control current employs a feed-forward compensator so a previous knowledge of the AC system is not necessary.

Based on equations (15) - (17), the phasor space equation is presented as:

\[ \mathbf{V}_{abc} - \mathbf{V}_{sabc} = L \frac{d\mathbf{i}_{abc}}{dt} + \mathbf{Ri}_{abc} \]  

(18)

Where \( \mathbf{V}_{abc} \) is the AC-side terminal three phase voltage vector, \( \mathbf{V}_{sabc} \) is the three phase voltage vector at the PCC point, \( i_{abc} \), is the vector current, \( L \), is the inductance of each phase, and, \( \mathbf{R} \), is the resistance of each phase.

As it was mentioned in section 2.1 the VSI operate based on switching six transistors S1-S6 through pulse with modulations SVPWM strategy. Since the VSI is considered a nonlinear time variant device, it is important to assume an average linear model to design the controller. Since it is known signal modulated from PWM can just take a value of 1 or -1, in order to design the controller, \( m_i \) is introduced as the average value of the signal modulated, which is going to be considered a sinusoidal function on average, then the terminals voltages can be expressed as:

\[
\begin{bmatrix}
V_{ta} \\
V_{tb} \\
V_{tc}
\end{bmatrix} = \frac{V_{dc}}{2} \begin{bmatrix}
m_a \\
m_b \\
m_c
\end{bmatrix}
\]  

(19)

Replacing the equation (19) in equations (15) to (17):

\[
m_a \frac{V_{dc}}{2} - V_{sa} = L \frac{d\mathbf{i}_a}{dt} + R_i_a
\]  

(20)

\[
m_b \frac{V_{dc}}{2} - V_{sb} = L \frac{d\mathbf{i}_b}{dt} + R_i_b
\]  

(21)

\[
m_c \frac{V_{dc}}{2} - V_{sc} = L \frac{d\mathbf{i}_c}{dt} + R_i_c
\]  

(22)

Then the space vector equation can be written as:

\[
m_{abc} \frac{V_{dc}}{2} - V_{sabc} = L \frac{d\mathbf{i}_{abc}}{dt} + \mathbf{Ri}_{abc}
\]  

(23)

In order to design the current controller in Alpha-Beta frame, the first step is transforming the system from the original reference frame to the stationary complex alpha-beta frame. This action is performed using the transformation matrix from equation (2) to equations (20) to (22), or from the space vector equation (23) and taking the real and imaginary part, so we have:

\[
m_a \frac{V_{dc}}{2} - V_{sa} = L \frac{d\mathbf{i}_a}{dt} + R_i_a
\]  

(24)

\[
m_b \frac{V_{dc}}{2} - V_{sb} = L \frac{d\mathbf{i}_b}{dt} + R_i_b
\]  

(25)
Figure 23 shows the average linear model of the plant. Based on equations (24) and (25). We know the plant transfer function is \( G(s) = \frac{1}{Ls + R} \), then, we can design the controller.

![Average Linear Model of the plant in Alpha-Beta Frame](image)

From equations (24) and (25) we know there are two independent systems so the control action is performed independently. Figure 24 shows the closed loop current control diagram in Alpha-Beta Frame. The reference currents \( i_{\alpha \text{ref}} \) and \( i_{\beta \text{ref}} \) are expected to be sinusoidal, so in order to track this periodic signals with small steady state error proportional resonant controller are used for this study. It is important to mention that the proportional resonant current controller could be implemented directly in the natural stationary frame, but the advantage of working in
Alpha-Beta frame is that we just need two controllers instead of three. Since the space vector, modulation strategy is implemented in Alpha-Beta coordinates, working in this vector space is also advantageous. From this figure, we also can see the feed forward compensator has the task to decouple between the VSI and the AC system.

4.2 Simulation of the Current Controller

The simulation of the system described in the last section was performed in PLEC. Figure 25 shows the two level 3 legs VSI model designed in this software, which consists of two IGBT transistors for each leg (six IGBT total). The control of each leg depends on one digital signal that excludes the possibility the two transistors of the same leg can be switched on at the same time by using a not gate as shown in this figure. The technique used to control the VSI is the two-dimensional space vector modulation presented in section 2.1.

From Figure 26 we can see the models of the power system and the current controller in alpha-beta frame developed in PLECS. The reference currents are compared with the VSI current, then the proportional resonant compensator can send the voltage signal required to follow the reference current through the space vector-PWM generator block, which in charge of controlling the VSI terminal voltages VSI. The space vector-PWM generator block is a C-script developed by PLECS and it is available in the software library in order to simulate this modulation technique.

Figure 25. Two level three phase voltage source inverter (3-legs) - PLECS Model
Figure 26. Closed Loop Current Control in α-β Frame with Feed Forward Compensator
PLECS Model (a) Power System (b) Controller

- **Simulation Example 3.2.1**

This simulation is based on the model of Figure 26. The three-leg VSI has the following parameters: $L = 100[\mu H]$, $R = 1.7[\Omega]$. The DC link voltage is $V_{dc} = 1000[V]$, and it is connected to the grid with a frequency $f = 60[Hz]$. The switching frequency is $f_s = 10[kHz]$ At the point of common coupling the voltage imposed by the grid is $V_S = 480[V]$ rms, which means that the peak value of the phase voltage is $V_{S_{peak}} = 391[V]$.

The current compensator used in $\alpha$-system and $\beta$-system of Figure 24 is the non-ideal proportional resonant compensator presented in section 3.1, which is described by equation (26). The proportional gain and the integral gain were chosen as it were a PI compensator and then adjusted in order to ensure a settle time of about 0.01 seconds and zero steady state error.

$$G_c = 1.5 + \frac{520s}{s^2 + 10s + (377)^2}$$  \hspace{1cm} (26)
From the above transfer function we can notice the complex poles of the controller at the AC system frequency in order to ensure a high gain at that frequency, therefore the zero steady state error is guaranteed at that specific frequency. The feed forward compensator employed is $G_c = 1/ (8 \times 10^{-6}s + 1)$.

Figure 27. Current Time Response (a-b-c Frame) – Current Controller

Figure 28. Current Time Response (Alfa-Beta Frame) – Current Controller
In this simulation the reference current is $i_{\text{ref}} = 1000\sin(2\pi f)$. Figure 27 shows the current time response of the system when the reference signal is applied, as we can see, the inverter currents $i_a, i_b$ and $i_c$ follow the reference current with zero steady state error, likewise from Figure 28 we can see that the currents $i_a$ and $i_b$ follow the reference current with zero steady state error in the Alpha-Beta frame, which means the controller is working properly.

### 4.3 Real and Reactive Power Open Loop Control / Simulation

Since the current is controlled and the model considered the voltages are known values and decoupled from the AC systems thanks to the feed forward compensator, it is possible to control the power send to the grid by though the current controller. Figure 29 shows the open loop real/reactive power control model designed in PLECS based on the current controlled presented in section 4.1. The grid voltage and current of each phase are measured in the system, then they are transformed to the alpha-beta frame, and the active and reactive power send to the system is calculated. From [1] we know the active and reactive power can be calculated in alpha-beta frame as:

$$ P_s = \frac{3}{2} \left[ V_{\alpha\alpha}(t)i_{\alpha}(t) + V_{\beta\beta}(t)i_{\beta}(t) \right] $$  \hspace{1cm} (27)  

$$ Q_s = \frac{3}{2} \left[ -V_{\alpha\alpha}(t)i_{\beta}(t) + V_{\beta\beta}(t)i_{\alpha}(t) \right] $$ \hspace{1cm} (28)

If the system is fast enough to approximate $P_s \approx P_s^*$ and $Q_s \approx Q_s^*$, therefore the reference current are derived from (27) and (28)

$$ \begin{bmatrix} i_{\alpha}^* \\\ n_{\beta}^* \end{bmatrix} = \frac{2}{3} \frac{1}{V_{\alpha\alpha}^2 + V_{\beta\beta}^2} \begin{bmatrix} V_{\alpha\alpha} & V_{\beta\beta} \\ V_{\beta\beta} & -V_{\alpha\alpha} \end{bmatrix} \begin{bmatrix} P_s^* \\ Q_s^* \end{bmatrix} $$ \hspace{1cm} (29)

Figure 29 shows the controller used for the open loop real/reactive power control, as we can see this model is based on the current controller from Figure 26, the difference now is that the current references are calculated by the signal reference generator through equation (29).
• **Simulation Example 3.3.1**

This simulation considers the same parameters of the simulation example 3.2.1, but now the current reference is imposed by the Signal Reference Generator according to the real/reactive power required to send to the grid. The dynamic response of the controllers is shown in Figure 30 and Figure 31. The active and reactive power time response is also shown in Figure 30, and the currents time response in Alpha-Beta is shown in Figure 31. As we can see the PWM controller is enabled at time $t=0.05[s]$ with zero active and reactive power. At time $t=0.1[s]$ the reference value of the reactive power is set to be $P^*=1$ [MW], as we can see, the real power reaches the reference value very fast and a little disturbance is reflected in the reactive power signal, but both signals follow the track perfectly the reference value. At time $t=0.2$ [s] a step change is imposed on the reference signal of the reactive power $Q^*=500$ [kVAR], as we can see the controller ensure a good a response to reach the new reference and track it with zero steady state error. Finally at time $t=0.3$ [s] a real power $P^*=-1$ [MW] is set which means a step change of -2 [MW], the controller react fast and follow the reference.

![Real Power Time Response](image1)

![Reactive Power Time Response](image2)

Figure 30. Real and Reactive Power Time Response – 3-Leg VSI

Figure 32 shows the phase voltage and the phase current $i_a$ at the point of common coupling. From this figure, we can see how the current change according to the required real and reactive power. Before $t=0.05$ [s] the system is not enabled, after this time the current is zero since the required values of the active and reactive power are zero, at time $t=0.1[s]$ the phase current $i_a$ is in phase with the phase $V_{sa}$ in order to send real power according to the requirements.
At time $t=0.2 \, [s]$ a step change is imposed the reference signal of the active power $Q^*=500 \, [kVAR]$ for which the phase current $i_a$ increases its peak value and become out of phase with respect to the voltage $V_{sa}$ as it expected, finally at time $t=0.3 \, [s]$ due to $P^*=-1 \, [MW]$ the current change the phase to send the required power, therefore we can realize the current controller and in general the real/reactive power controller are working with high performance according to the design requirements.
4.4 Real and Reactive Power Closed Loop Control / Simulation

The closed loop real and reactive power control consist on calculating the power send to the AC system with the currents and voltages measured at the PCC and comparing it with the reference values of power. This type of control is better done in the d-q frame since the control of the real power and the reactive power is decoupled.

From [1] we know the active and reactive power can be calculated in the rotating d-q frame through the following equations:

\[
P_s = \frac{3}{2} [V_{sd}(t)i_d(t) + V_{sq}(t)i_q(t)] 
\]

\[
Q_s = \frac{3}{2} [-V_{sd}(t)i_q(t) + V_{sq}(t)i_d(t)] 
\]

Now, if the vector voltage at the point of common coupling \( V_s \) is aligned with the direct axis, the voltage component in the q-axis will be zero and the equations (30) and (31) become:

\[
P_s = \frac{3}{2} V_{sd}(t)i_d(t) 
\]

\[
Q_s = -\frac{3}{2} V_{sd}(t)i_q(t) 
\]

From (32) and (33) we can see that the real power is proportional to the \( i_d \) current, and the reactive power is proportional to \( i_q \) current, so the control action is decoupled which is not possible using any stationary frame. If we consider that the control current is fast enough, the transfer function of the plant described by equations (34) and (35). As we can see the plant is just a constant, so to control both real and reactive power a PI compensator is enough.

\[
\frac{i_d}{P_s} = \frac{2}{3} \frac{1}{V_d} 
\]

\[
\frac{i_q}{Q_s} = -\frac{2}{3} \frac{1}{V_d} 
\]

The real/reactive power controller modelled in PLECs is shown in Figure 33. As we can see this controller is also based on the current controlled presented in section 4.1, which works in Alpha-Beta, since the control action is taking place in d-q frame, the reference signals generated by the PI controller has to be transformed to Alpha-Beta as shown in this figure.
Simulation Example 3.3.1

This simulation has the same parameter of the simulation performed in section 4.2, but now instead of having a reference signal generator, PI controllers in dq frame are employed, see Figure 33. We have a compensator to regulate the dynamic response of the real and reactive power. The PI compensator of the real power is $G_p = (10^{-3}s + 11)/s$, and the PI compensator of the reactive power is $G_q = -(10^{-3}s + 11)/s$, which were designed in order to get a settle time of 20 milliseconds.

Figure 34 shows the active and reactive power time response, as we can see, the real al reactive power responses are faster than in the simulation developed in section 4.2 due to the PI compensators. From Figure 35 we can see a faster and strongest current response than the signal from Figure 32, at the moment when active or reactive power is changed.
Figure 34. Voltage (VSa) and Current (Ia) Time Response—Phase A

Figure 35. Voltage (VSa) and Current (Ia) Time Response—Phase A
4.5 LC Filter

In this section, a model of the Grid Connected VSI with an LC Filter is presented.

Figure 36 shows the topology of a grid connected voltage source inverter with an LC filter, that interfaces the VSI terminals with the point of common coupling (PCC), and an LCL filter that interfaces the VSI with the main voltage supply. The presence of this capacitor is important to maintain the voltage level at PCC, also it provides a low impedance path in order to prevent current harmonics propagate into the AC system.

The LCL filter has the following components: \( Z_L = L_s \), \( Z_C = \frac{1}{sC} \), \( Z_g = L_g s \)

In order to design the current controller for this system we can calculate the following transfer functions:

\[
\frac{I(s)}{V_i(s)} = \frac{Z_g + Z_C}{Z_L Z_g + Z_C Z_g + Z_L Z_C} \tag{36}
\]

\[
\frac{I_s(s)}{V_i(s)} = \frac{Z_g}{Z_L Z_g + Z_C Z_g + Z_L Z_C} \tag{37}
\]

Where \( I \), is the output current of the voltage source inverter, \( I_s \), is the output current send to the AC system and \( V_i \) is the voltage at terminals of the VSI. From these two transfer function we realized that two possible control actions can be implemented, the first one is controlling the output current of the voltage source inverter, and the second one is controlling the output current send to the AC system.

The Bode Plot of the equation (36) is shown in Figure 37, where we can see the frequency characteristic of \( I(s)/V_i(s) \) of the LCL filter. It can be noticed that this filter behaves as an equivalent L-filter model (\( L + L_s \)) for frequencies lower and higher to the resonant frequency.
which means that for those frequencies we can apply the same theory and design methods treated in the last sections 4.1-4.4 in order to design the current controller.

**Simulation Example 4.1.1**

This simulation is based on the model of Figure 36. The three-leg VSI has the following parameters: \( L = 100[\mu H], R = 1.7[m\Omega], \ C = 250[\mu F], \ L_g = 50[\mu H]. \) The DC link voltage is \( V_{dc} = 1000[V], \) and it is coned to the grid with a frequency \( f = 60 [Hz] \). The switching frequency is \( f_s = 10[kHz]. \) The supply voltage \( V_s = 480[V] \)

As we mention before the implemented control strategy in this simulation is the same as in the sections 4.2 and 4.3. The proportional current compensators employed in the \( \alpha \beta - \) control system is:

\[
G_c = 0.11 + \frac{75s}{s^2 + 30s + (377)^2}
\]  

(38)

The feed forward compensator employed is \( G_c = 1/(8\times10^{-6}s+1). \)

In this simulation, the open loop real/reactive power control described in section 4.3 was employed. Since the current controlled is not the VSI output current the signal reference generator take into account the reactive power send to the system by the capacitor.

Figure 38 shows the active and reactive power time response of this simulation, as we can see the real and reactive power signals don’t have the switching effect cause by the VSI at PCC due to the presence of the capacitor. From this figure, we can see that although the power signals follow the reference with zero stable state error, the real and reactive power are influenced by each other during transient states, at the moment of changing the reference values. This behaviour is due to the current controller was developed in the stationary Alpha-Beta frame, so a transient state in the alpha or beta component of the current will affect both real/reactive power. As we mentioned before the unique way to avoid the mutual influence of
the real/reactive power during transient states is using a controller completely in the synchronous d-q frame, where we can make the real power proportional to the current in the direct axis, and the reactive power proportional to the current in the quadrature axis.

Figure 38. Real and Reactive Power Time Response – LC Filter
5. Voltage Control – Controlled Frequency 3-Leg VSI System

5.1 Voltage Control Design in Alpha-Beta Frame

In this section is presented the control of the voltage across the capacitor at the point of common coupling when the frequency of the voltage is not imposed by the grid. This could be the case when the VSI operate in a micro-grid or stand alone. The low impedance path provided by the capacitor for switching prevent current harmonics propagate into the AC system/load.

The voltage control action is performed through the use of the current controller presented in section 4.1. For this model it is assumed that the DC voltage is an ideal power supply. The VSI uses a RLC filter to interface with the AC system/Load. The dynamic of the AC system / Load is not being considered since the control current employs a feed-forward compensators so a previous knowledge of the AC system/Load is not necessary.

![Figure 39. Schematic diagram of the 3 legs VSI Voltage Control System](attachment:image.png)

From Figure 39 we can say that the dynamic model of the voltage at the point of common coupling PCC is described by the following state space equation:

$$
C \frac{d}{dt} \begin{bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \begin{bmatrix} i_{La} \\ i_{Lb} \\ i_{Le} \end{bmatrix}
$$

(39)

Transforming the equation (39) to the Alpha-Beta stationary frame using the equation (2), we have:
\[ C \frac{d}{dt} \begin{bmatrix} V_{Sa} \\ V_{S\beta} \end{bmatrix} = \begin{bmatrix} i_\alpha \\ i_\beta \\ i_{L\alpha} \\ i_{L\beta} \end{bmatrix} \] \quad (40)

Using the current control developed in section 4.1, the dynamic model of the closed loop current control system can be described by the equations:

\[ i_\alpha(s) = G_{PC\alpha}(s)i_{ref\alpha} \quad (41) \]
\[ i_\beta(s) = G_{PC\beta}(s)i_{ref\beta} \quad (42) \]

Where, \( G_{PC\alpha}(s) \) is the transfer function between the controlled current \( i_{\alpha\beta}(s) \) and reference value \( i_{\alpha\beta\text{ref}} \). The unknown AC system or load, represented by the terms \( i_{L\alpha}, i_{L\beta} \) in equation (40) can be formulated as:

\[ \begin{bmatrix} i_{L\alpha} \\ i_{L\beta} \end{bmatrix} = \begin{bmatrix} f(x_1, x_2, ..., x_n, V_{Sa}, t) \\ g(x_1, x_2, ..., x_n, V_{S\beta}, t) \end{bmatrix} \quad (43) \]

Where \( V_{Sa} \) and \( V_{S\beta} \) are the inputs of this system, and \( x_1, x_2, ..., x_n \) are the state variables that define the system by the non-linear functions \( f(\bullet) \) and \( g(\bullet) \). But, as we mentioned before instead of considering the dynamic model of the load, a feed-forward strategy is used.

Figure 40 shows the dynamic model of the system in Alpha-Beta frame based on equations (40) - (43).

![Dynamic model of the voltage at the point of common coupling - Alpha Beta Frame](image)

Figure 40. Dynamic model of the voltage at the point of common coupling - Alpha Beta Frame

40
The closed loop voltage control system is based on the models shown in Figure 41, the voltages $V_{S\alpha}$, and $V_{S\beta}$ are controlled through $i_{ref\alpha}$, and $i_{ref\beta}$. These two systems are decoupled so they are controlled independently. Since the dynamics of the load is unknown it is important to measure the load currents $i_{L\alpha}$, and $i_{L\beta}$, in order to include their impact on the control action as another strategy of feedforward compensation. The voltage compensator is a proportional resonant controller which have to track a sinusoidal voltage reference.

![Figure 41. Closed loop Voltage Control Diagram - Alpha Beta Frame](image.png)

5.2 Simulation of the Voltage Controller

From Figure 42 (a) and (b) we can see the models of the power system and the voltage controller in Alpha-Beta frame developed in PLECS for autonomous operation. The power system consists of three-leg voltage source inverter connected to a RLC branch, two loads are attached at the point of common coupling in this model. The voltage control action is implemented through the current controller as shown in this figure, the reference voltage is compared with the VSI voltage measured at the point of common coupling, then the proportional resonant compensator send the current signal reference in order to perform the current control action.
From Figure 42(b), we can see that the magnitude and frequency of the voltage reference signal are generated through a dq transformation, where Vd represent the magnitude of the voltage reference signal since Vq is imposed to be zero, which means the vector reference voltage is aligned with the d-axis. The frequency is set up as a constant value and integrated in order to get the angle that we require to perform the frame transformation.

Three different simulations were carried out in order to test the closed loop voltage control, the first simulation test the voltage changes when there is no load, the second simulation is taking place when a load is considered, and the third simulation tests the voltage control when different loads are suddenly connected to the system.
Simulation Example 5.2.1

This simulation is based on the power system shown in Figure 42 (a), and consider the following parameters for the RLC branch: \( L = 100[\mu H] \), \( R = 1.7[m\Omega] \), and \( C = 2500[\mu F] \). The DC link voltage is \( V_{DC} = 1000[V] \). The switching frequency is \( f_s = 10[kHz] \).

The first load attached to the system is: \( R_1=150[mOhm] \) and \( L_1=80[uH] \). The second load attached to the system is: \( R_2=100[mOhm], L_2=70[uH] \) and \( C_2=10[mF] \).

The voltage compensator used in the \( \alpha \)-control system and \( \beta \)-control system from Figure 41 is the non-ideal proportional resonant compensator presented in section 3.1, and is described by the following transfer function.

\[
G_s = 7.5 + \frac{2500s}{s^2 + 10s + (377)^2}
\]  

(44)

As in section 4.2 the proportional gain and the integral gain of this compensator were chosen as the controlled were a PI controlled and then adjusted in order to ensure a settle time of about 0.025 seconds and zero steady state error.

For this simulation, the magnitude of the reference voltage \( V_{d\_ref} \) is stepped in order to test the voltage controller. The frequency is imposed to be \( f = 60[Hz] \). This simulation, is performed at no load conditions, so the switches that joint the loads with the system are never closed. Figure 43, shows the PCC voltage time response in the rotary frame dq, and in the stationary frame a-b-c, and the real and reactive power send to the system. The voltage reference is changed from zero to 400 [V] at time \( t=0[s] \), this state is maintained until the time \( t=0.1[s] \), where the magnitude of the reference voltage is changed from 400 [V] to 450 [V], and finally the reference voltage is return to being 400 [V] at the time \( t=0.2[s] \). In this figure, we can see that the voltage measured at the point of common coupling takes around 0.02 seconds to reaching the stable state and tracks the reference signal with zero steady state error which means the voltage compensator of equation (44) is working properly. Since the VSI is working at no-load the real and reactive power send to the system is zero as we can see in this figure.
Simulation Example 5.2.2

This simulation is based on the power system shown in Figure 42 (a). This example employs same parameters and compensators of the simulation performed in the simulation example 5.2.1. The reference voltage signal is also the same but now the switch that connects the PPC with the Load 1 is closed. The frequency is imposed to be $f = 60 \text{ [Hz]}$. 

Figure 43. PPC Voltage and Power Dynamic Response (No-Load) 3-Leg VSI
Figure 44. PPC Voltage and Power Dynamic Response (Load-1) 3-Leg VSI
From Figure 44, we can see the PCC voltage time response in the rotary frame dq, and in the stationary frame a-b-c, and the real and reactive power send to the system. Likewise in the last simulation example, the voltage reference is changed from zero to 400 [V] at time t=0 [s], this state is maintained until the time t=0.1 [s], where the magnitude of the reference voltage is changed from 400 [V] to 450 [V], and finally the reference voltage is return to being 400 [V] at the time t=0.2 [s]. In this figure, we can see that the voltage measured at the point of common coupling takes around 0.021 seconds to reaching the stable state and tracks the reference signal with zero steady state error. Unlike the last simulation, the active and reactive power send to the system is not zero, as we can see the active power is about 1.5 megawatts when the voltage is at 400 [V] and about 2 megawatts when the voltage is 450 [V]. The reactive power is 300 [VAR] and 400 [VAR] respectively.

- **Simulation Example 5.2.3**

The objective in this simulation was to test the voltage controller when it is requested to maintain a constant voltage and different loads are connected to the system.

This simulation is based on the power system shown in Figure 42 (a) and has the same parameters and compensators of the simulation example 5.2.1. So, the magnitude of the reference voltage signal is maintained at a constant value of 400 [V]. The frequency imposed for this simulation was \( f = 60 \text{ [Hz]} \). At time t=0 [s] there is no load, at time t=0.1 [s] the Switch 1 is closed in order to connect the Load 1, and finally at time t=0.2 [s] the Switch 2 is closed to connect the Load 2.

Figure 45 shows the time response of the PCC voltage in the rotary frame dq, and in the stationary frame a-b-c. The real and reactive power send to the system are also shown. As we can see the voltage controller is capable of maintaining the voltage level, but at the moment of load’s connection, there are small transient states that go quickly to the stable state. The power that is sent to the system is going to change in time due to the loads added to the system, therefore from time t=0 [s] to time t=0.1 [s] the active and reactive power sent to the system are zero as we can see from Figure 45, afterward the system send positive real and reactive power due to the presence of Load 1, at time t=0.2 [s] due to the presence of Load 2 the real power send to the system gets higher and the reactive power becomes negative.
Figure 45. PPC Voltage and Power Dynamic Response (Load 1 & Load 2)
6. Four-leg voltage source inverter, modelling, and control

6.1 Current Control Design in Alpha-Beta-Gamma Frame

The average large signal model of the two level 4 legs voltage source inverter considered for the current control design is based on the schematic diagram of the circuit shown in Figure 46, which is described by the following equations, it is important to mention that for this model is assumed that DC link is connected to an ideal DC voltage source:

\[ V_{af} = L \frac{di_a}{dt} + Ri_a + V_{sa} - \frac{di_n}{dt} L_n \]  \hspace{1cm} (45)

\[ V_{bf} = L \frac{di_b}{dt} + Ri_b + V_{sb} - \frac{di_n}{dt} L_n \]  \hspace{1cm} (46)

\[ V_{cf} = L \frac{di_c}{dt} + Ri_c + V_{sc} - \frac{di_n}{dt} L_n \]  \hspace{1cm} (47)

\[ i_a + i_b + i_c = -i_n \]  \hspace{1cm} (48)

Where \( V_{af} \) is the voltage between the terminal voltage \( V_{sa} \) and the terminal voltage \( V_{tf} \), the voltage \( V_{bf} \) is the voltage measured between the terminal voltage \( V_{sb} \) and the terminal voltage \( V_{tf} \), and \( V_{cf} \) is the voltage between the terminal voltage \( V_{tc} \) and the terminal voltage \( V_{tf} \). These terminal voltages at the VSI can be considered as controlled voltage sources described by equation 48.
(49), \( m_i \) is introduced as the average value of the signal modulated, which is going to be considered a sinusoidal function on average

\[
\begin{bmatrix}
V_{af} \\
V_{bf} \\
V_{cf}
\end{bmatrix} = V_{DC} \begin{bmatrix}
m_{af} \\
m_{bf} \\
m_{cf}
\end{bmatrix}
\] (49)

The voltage at the point of common coupling PCC point are \( V_{sa} \), \( V_{sb} \) and \( V_{sc} \) are the voltages at (PCC), and \( \frac{d}{dt} L_n \) is the voltage across the inductor \( L_n \). Since the control current employs a feed-forward compensator to decouple the AC system from the voltage source inverter the voltages at the PCC point are considered known values or grid imposed voltages

The dynamic equation of this model is:

\[
\frac{d}{dt} \begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} = V_{DC} \frac{1}{L} \begin{bmatrix}
m_a \\
m_b \\
m_c
\end{bmatrix} - R \begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} - \frac{1}{L} \begin{bmatrix}
V_{sa} \\
V_{sb} \\
V_{sc}
\end{bmatrix} + \frac{L_n}{L} \frac{d}{dt} \begin{bmatrix}
i_n
\end{bmatrix}
\] (50)

\[
i_a + i_b + i_c + i_n = 0
\] (51)

From this equations we can see that this model has three subsystems that are coupled through the neutral current, so the control action cannot take separate for each system in order to have three different system to be controlled independently, this model is transformed to the Alpha-Beta-Gamma stationary frame thought the transformation matrix from equation (6). So:

\[
\frac{d}{dt} \begin{bmatrix}
i_\alpha \\
i_\beta \\
i_\gamma
\end{bmatrix} = V_{DC} \frac{1}{L} \begin{bmatrix}
m_\alpha \\
m_\beta \\
m_\gamma
\end{bmatrix} - R \begin{bmatrix}
i_\alpha \\
i_\beta \\
i_\gamma
\end{bmatrix} - \frac{1}{L} \begin{bmatrix}
V_{s\alpha} \\
V_{s\beta} \\
V_{s\gamma}
\end{bmatrix} + \frac{3L_n}{L} \frac{d}{dt} \begin{bmatrix}
0 \\
0 \\
i_\gamma
\end{bmatrix}
\] (52)

As we can see from equation (52), now three are three independent subsystem that are no coupled, so the control action is independent for each one. Figure 47 shows the average linear model of the plant based on equations (23) and (24). We know the plant transfer function is \( G(s) = \frac{1}{Ls + R} \), then, we can design the controller.
6.2 Simulation of the Current Controller

Figure 49 shows the two level 4-leg voltage source inverse model designed in PLECS. This model that consists of two IGBT transistors for each leg, eight total. As the 3-leg VSI, the control of each leg depends on one digital signal that excludes the possibility the two transistors of the same leg can be switched on at the same time by using a not gate as shown in this figure. The technique used to control the VSI is the three-dimensional space vector modulation presented in section 2.2

The power system and the current controller in alpha-beta-gamma frame developed in PLECS are shown in Figure 50. The reference currents are compared with the VSI current then the proportional resonant compensator sends the voltage required to track the reference current through the 3D space vector-PWM generator block, which is in charge of controlling the VSI terminal voltages. The 3D space vector-PWM generator block is a C-script, which was implemented in section 2.3 in order to simulate this modulation technique. It is important to notice that all the signals are converted to alpha-beta-gamma frame before to enter into the controller.
Figure 48. Closed Loop Current Control Block Diagram - Alpha-Beta-Gamma Frame
Figure 49. Two level three phase 4-legs VSI – PLECS model

Figure 50. Closed Loop Current Control in $\alpha$-$\beta$-$\gamma$ Frame with Feed Forward Compensator-PLECS Model (a) Power System (b) Controller
Simulation Example 5.2.1

This simulation of the closed loop current control, is based on the model of Figure 50(a). The parameters and conditions considered in this simulation are the same as the current controller simulation of section 4.2, but now, instead of using the 3-leg VSI, the 4-leg VSI is being used. The parameters are: \( L = 100[\mu H] \), \( L_n = 100/3[\mu H] \), \( R = 1.7[m\Omega] \). The VSI is connected to the grid with a frequency \( f = 60 [HZ] \). The switching frequency is \( f_s = 10[kHz] \). At the point of common coupling, the voltage imposed by the grid is \( V_S = 480[V] \) rms, which means that the peak value of the phase voltage is \( V_{S\phi,peak} = 391[V] \). So in order to have DC link voltage we consider at least \( \sqrt{3}V_{S\phi,peak} \). The DC link voltage is \( V_{DC} = 1000[V] \).

The compensators used in \( \alpha \)-system and \( \beta \)-system of Figure 48 (a) and (b) is the same non-ideal proportional resonant compensator used with the 3-leg inverter, which is described by equation (53). The proportional gain and the integral gain were chosen as explained in section 4.2

\[
G_{c-\alpha\beta} = 1.5 + \frac{520s}{s^2 + 10s + (377)^2} \quad (53)
\]

The compensators used in gamma-system is the non-ideal proportional described by the transfer function bellow:

\[
G_{c-\gamma} = 3 + \frac{1040s}{s^2 + 10s + (377)^2} \quad (54)
\]

The complex poles of the controllers at the AC system frequency ensure a high gain at that frequency in order to track the reference signal with zero steady state error. The feed forward compensator employed is \( G_c = 1/(8 \times 10^{-6} s + 1) \).

The reference current signal used in this simulation is \( i_{ref} = 1000 \sin(2\pi f) \). Figure 51 shows the current time response, as we can see, the inverter currents \( i_a, i_b \) and \( i_c \) follows the reference current with zero steady state error, likewise from Figure 52 we can see that the currents \( i_a \) and \( i_\beta \) follow the reference current with zero steady state error in the Alpha-Beta-Gamma frame, since this simulation was developed under balance current reference it is logic that the current \( i_\gamma \) follow a zero reference current.
Figure 51. Current Time Response (a-b-c Frame) – Balanced Current Reference

Figure 52. Current Time Response (Alfa-Beta-Gamma Frame) – Balanced Condition
Simulation Example 5.2.2

An advantage of using a 4-leg VSI over a 3-leg VSI is we can deal with the zero sequence currents. This simulation considers the same system and parameters of the last simulation. The proportional resonant controllers are also the same. But now the reference currents are:

\[
\begin{align*}
    i_{a\text{ ref}} &= 1000\sin(377t)\,[A] \\
    i_{b\text{ ref}} &= 800\sin(377t - \frac{\pi}{3})\,[A] \\
    i_{c\text{ ref}} &= 100\sin(377t + \frac{2\pi}{3})\,[A]
\end{align*}
\]

Figure 53 shows the current time response. As we can see, the inverter currents \(i_a\), \(i_b\), and \(i_c\) follows the reference current with zero steady state error, likewise from Figure 54 we can see that the currents \(i_\alpha\), \(i_\beta\), and \(i_\gamma\) follow the reference current with zero steady state error in the Alpha-Beta-Gamma frame, since this simulation was developed under unbalanced conditions the reference current in the gamma axis is a sinusoidal signal, which is proportional to the zero sequence of the current.
The real and reactive power open loop control for the four-leg voltage source inverter is developed under the same criteria used in section 4.3, which is controlling the power through the current controller, so the reference currents are calculated using the known values of the voltage at the point of coming coupling and the real and reactive power reference values.

Figure 55 shows the open loop real/reactive power control model is designed in PLECS based on the current controlled presented in section 6.1. From [4] we know the active and reactive power can be calculated using equations (55) and (56)

![Current Time Response (Alpha-Beta-Gamma Frame)](image1)

![Current Time Response (Alpha-Beta-Gamma Frame)](image2)

![Current Time Response (Alpha-Beta-Gamma Frame)](image3)

Figure 54. Current Time Response (Alfa-Beta-Gamma Frame) – Unbalanced Currents

### 6.3 Real and Reactive Power Open Loop Control / Simulation

The real and reactive power open loop control for the four-leg voltage source inverter is developed under the same criteria used in section 4.3, which is controlling the power through the current controller, so the reference currents are calculated using the known values of the voltage at the point of coming coupling and the real and reactive power reference values.

Figure 55 shows the open loop real/reactive power control model is designed in PLECS based on the current controlled presented in section 6.1. From [4] we know the active and reactive power can be calculated using equations (55) and (56)
\[ p(t) = V_	ext{abcS} \cdot i_	ext{abc} \quad (55) \]
\[ q(t) = V_	ext{abcS\perp} \cdot i_	ext{abc} \quad (56) \]

Where, \( V_	ext{abcS} \) is the vector voltage and \( i_	ext{abc} \) is the vector current measured at the point of common coupling. The \( V_	ext{abcS\perp} \) is the orthogonal vector voltage defined in equation (57)

\[
\begin{bmatrix}
V_{a\perp} \\
V_{b\perp}
\end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix}
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix} \begin{bmatrix}
V_a \\
V_b
\end{bmatrix} \quad (57)
\]

From the power theory described in [4] and considering equations (55) and (56) we know the current vector aligned with the vector voltage means pure active power, while any vector current aligned with the orthogonal vector voltage means pure reactive power. These currents can be represented by the equations:

\[ i_p = C_p V_	ext{abcS} \quad (58) \]
\[ i_q = C_q V_	ext{abcS\perp} \quad (59) \]

where \( i_p \), is the active current vector and \( i_q \), is the reactive current vector. \( C_p \) and \( C_q \) are proportional coefficients. Then:

\[ P = V_	ext{abcS} \cdot i_p = C_p V_	ext{abcS} \cdot V_	ext{abcS} = C_p |V|^2 \quad (60) \]
\[ Q = V_	ext{abcS\perp} \cdot i_q = C_q V_	ext{abcS\perp} \cdot V_	ext{abcS\perp} = C_q |V|^2 \quad (61) \]

From these two equations, the coefficients of the current vectors can be derived as:

\[ C_p = \frac{P}{|V|^2} \quad (62) \]
\[ C_q = \frac{Q}{|V|^2} \quad (63) \]

If the system is fast enough to approximate \( P_r \approx P_r^* \) and \( Q_r \approx Q_r^* \), the reference current can be derived from equations (58), (59), (62), and (63)

\[ i_p^* = \frac{P^*}{|V|^2} V_	ext{abcS} \quad (64) \]
\[ i_q^* = \frac{Q^*}{|V|^2} V_	ext{abcS\perp} \quad (65) \]

The total reference current will be
Finally, in order to send the reference current, these have to converter from the \( a-b-c \) frame to the \( \text{Alpha-Beta-Gamma} \) frame as we can see in Figure 55.

### Simulation Example 6.3.1

This simulation considers the same parameters of the simulation example 6.2.1, but now the current reference is imposed by the Signal Reference Generator. The active and reactive power time response is shown in Figure 56, and the currents time response in alpha-beta-gamma frame is shown in Figure 57. Since the system is balanced the current in the gamma-axis is zero. We can see the PWM controller is enabled at time \( t=0.05 \text{[s]} \), sending zero active and reactive power. At time \( t=0.1 \text{[s]} \) the reference value of the reactive power is set to be \( P^*=1 \text{[MW]} \), as we can see, the real power reaches the reference value very fast and a little disturbance is reflected in the reactive power signal, but both signals track perfectly the reference value. At time \( t=0.2 \text{[s]} \) a step change is imposed on the reference signal of the reactive power \( Q^*=500 \text{[kVAR]} \). Finally at time \( t=0.3 \text{[s]} \) a real power \( P^*=-1 \text{[MW]} \) is set which means a step change of \(-2 \text{[MW]} \). From this simulation we can see the performance of this control is equivalent to the 3-leg VSI case shown in Simulation Example 3.3.1.
Figure 56. Real and Reactive Power Time Response – 4 Leg VSI

Figure 57. Current Time Response (Alfa-Beta-Gamma Frame) – 4-Leg VSI
From the simulations of the 4-leg inverter we know that in a balanced system the control action in the gamma axis makes the values of the voltage and current equal to zero and the performance of the 3-leg VSI and 4-leg VSI are equivalent.

6.4 Real and Reactive Power Closed Loop Control / Simulation
The theory and the performance of the closed loop real and reactive power control under balanced conditions for the 4-Leg VSI is the same as the one presented in the section 4.4, for that reason simulations examples are not presented in this section.
7. **Voltage Control - Controlled Frequency 4-Leg VSI System**

7.1 **Voltage Control Design in Alpha-Beta-Gamma Frame**

In this section, we study the control of the voltage across the capacitor at the point of common coupling when the voltage source converter system is working in autonomous operation, so the frequency of the voltage is not imposed by the grid.

As it was mentioned before, the low impedance path provided by the capacitor prevents the current harmonics from propagating into the Load.

The voltage control action is performed through the use of the current controller presented in section 6.1. For this model, it is assumed that the DC voltage is an ideal power supply. The VSI uses a RLC filter to interface with the AC system/Load. The dynamic of the AC system/Load is not being considered since the control current employs a feed-forward compensators so a previous knowledge of the AC system/Load is not necessary.

![Figure 59. Schematic diagram of the 4-leg VSI Voltage Control System](image)

From Figure 59 we can say that the dynamic model of the voltage at the point of common coupling PCC is described by the following state space equation:

\[
C \frac{d}{dt} \begin{bmatrix} V_{Sa} \\ V_{Sb} \\ V_{Sc} \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \begin{bmatrix} i_{La} \\ i_{Lb} \\ i_{Lc} \end{bmatrix} \quad (67)
\]
Figure 60. Dynamic model of the voltage at the point of common coupling PCC (Alpha-Beta-Gamma Frame)

Transforming the equation (67) to the Alpha-Beta-Gamma stationary frame using the transformation matrix of equation (6) we have:
\[
\frac{d}{dt}\begin{bmatrix} V_{sa} \\ V_{s\beta} \\ V_{sy} \end{bmatrix} = \frac{1}{C} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_\gamma \end{bmatrix} - \frac{1}{C} \begin{bmatrix} i_{La} \\ i_{L\beta} \\ i_{Ly} \end{bmatrix}
\] (68)

Using the current control developed in section 6.1, we know that the dynamic model of the closed loop current control system can be described by the equations:

\[
i_\alpha(s) = G_{PCa}(s)i_{\alpha ref} \quad (69)
\]
\[
i_\beta(s) = G_{PC\beta}(s)i_{\beta ref} \quad (70)
\]
\[
i_\gamma(s) = G_{PC\gamma}(s)i_{\gamma ref} \quad (71)
\]

where, \( G_{PCa}(s) \) is the transfer function between the controlled current \( i_{\alpha ref} \) and reference value \( i_{\alpha ref} \). The unknown load, represented by the terms \( i_{La} \), \( i_{L\beta} \) and \( i_{Ly} \) in equation (68) can be formulated as:

\[
\begin{bmatrix} i_{La} \\ i_{L\beta} \\ i_{Ly} \end{bmatrix} = \begin{bmatrix} f(x_1, x_2, \ldots, x_n, V_{sa}, t) \\ g(x_1, x_2, \ldots, x_2, V_{s\beta}, t) \\ h(x_1, x_2, \ldots, x_n, V_{sy}, t) \end{bmatrix}
\] (72)

where \( V_{sa}, V_{s\beta} \) and \( V_{sy} \) are the inputs of this system, and \( x_1, x_2, \ldots, x_n \) are the state variables that define the system by the non-linear functions \( f(\bullet), g(\bullet), \) and \( h(\bullet) \). But, as we mentioned before instead of consider the dynamic model of the load, a feed-forward strategy is used. Figure 60 shows the dynamic model of the system in Alpha-Beta frame based on equations (68)-(72)

The closed loop voltage control system is based on the models shown in Figure 61, the voltages \( V_{sa}, V_{s\beta} \), and \( V_{sy} \) are controlled through \( i_{\alpha ref} \), \( i_{\beta ref} \), and \( i_{\gamma ref} \). These three systems are decoupled so they are controlled independently. Since the dynamics of the load is unknown it is important to measure the load currents \( i_{La}, i_{L\beta}, \) and \( i_{Ly} \) in order to include their impact on the control action as another strategy of feedforward compensation. Since the model was developed in the \( \alpha-\beta-\gamma \) stationary frame, the voltage compensators have to be proportional resonant controllers.
From Figure 62 (a) and (b) we can see the models of the power system and the voltage controller in Alpha-Beta-Gamma frame developed in PLECS for the PCC voltage control (autonomous operation). The power system consists of four-leg voltage source inverter.
connected to a RLC branch, two loads are attached at the point of common coupling in this model. The voltage control action is implemented through the current controller as shown in this figure. The reference voltage is compared with the VSI voltage measured at the point of common coupling, then the proportional resonant compensator send the reference current signal in order to perform the current control action, that was described in section 6.1

The magnitude and frequency of the voltage reference signal can be generated through a dq transformation as it was performed in section 5.2, if we want balanced voltage reference values, or from a signal generator in the \( a-b-c \) frame, then using a converter to get the voltage reference signal is the \( \alpha-\beta-\gamma \) stationary frame as we can see in Figure 62 (b)
The same three simulations performed in section 5.2 were carried out in this section in order to test the closed loop voltage control in the alpha-beta-gamma reference frame.

- **Simulation Example 7.2.1**

This simulation is based on the power system shown in Figure 62 (a). This simulation considers the following parameters for the RLC branch: \( L = 100[\mu H] \), \( R = 1.7[m\Omega] \), and \( C = 2500[\mu F] \). The DC link voltage is \( V_{DC} = 1000[V] \). The switching frequency is \( f_s = 10[kHz] \) the value of inductance connected to the fourth leg is: \( L_n = 100/3[\mu H] \).

The first load attached to the system is: \( R_1=150[mOhm] \) and \( L_1=80[uH] \). The second load attached to the system is: \( R_2=100[mOhm] \), \( L_2=70[uH] \) and \( C_2=10[mF] \)

The voltage compensator used in the \( \alpha \)-control system and \( \beta \)-control system shown in Figure 61 is the same non-ideal proportional resonant compensator presented in section 5.2. For the \( \gamma \)-control system the compensator function is shown below:

\[
G_{\gamma} = 5 + \frac{2400s}{s^2 + 10s + (377)^2} \tag{73}
\]

The proportional gain and the integral gain of this compensator were tuned from the \( \alpha \beta \) voltage compensator described by the transfer function of equation (44) (see section 5.2)

For this simulation, the magnitude of the reference voltage \( V_d \) is stepped in order to test the voltage controller. The frequency is imposed to be \( f = 60[Hz] \). This simulation is performed at no load conditions, so for these simulations, the switches that joint the loads with the system are never closed. Figure 43, shows the PCC voltage time response in the rotary frame dq, and in the stationary frame a-b-c, and the real and reactive power send to the system. The voltage reference is changed from zero to 400 [V] at time \( t=0[s] \), this state is maintained until the tome \( t=0.1[s] \), where the magnitude of the reference voltage is change from 400 [V] to 450 [s], and finally the reference voltage is return to being 400 [V] at the time \( t=0.2[s] \). In this figure, we can see, that in the 3-leg VSI case, the voltage measured at the point of common coupling takes around 0.02 seconds to reaching the stable state and tracks the reference signal with zero steady state error which means the voltage compensators are working properly. Since the VSI is working at no-load the real and reactive power send to the system is zero as we can see in this figure. From the simulation of this balanced system, we can say that the performance of the voltage controller using 4-leg VSI is the same as using the 3-leg VSI.
Simulation Example 7.2.2

This simulation is based on the power system shown in Figure 62(a). This simulation employs the same parameters and compensators of the simulation performed in the Simulation Example 7.2.1. The reference voltage signal is also the same but now the Load 1 is connected to the system. The frequency is imposed to be $f = 60 \, [Hz]$.

Figure 64, shows the PCC voltage time response in the rotary frame dq, and in the stationary frame a-b-c, and the real and reactive power send to the system. Since the results of this
simulation are equivalent to the Simulation Example 5.2.2 performed with the three-leg voltage source inverter, the same analysis can be applied to this specific example.

Figure 64. PPC Voltage and Power Dynamic Response (Load-1) 4-Leg VSI
The objective in this simulation was to test the voltage controller when it is requested to maintain a constant voltage and different loads are connected to the system.

This simulation is based on the power system shown in Figure 62 (a) and has the same parameters and compensators of the simulation example 7.2.1. So, the magnitude of the reference voltage signal is maintained at a constant value of 400 [V]. The frequency imposed for this simulation was \( f = 60 \, [Hz] \). At time \( t=0 \, [s] \) there is no load, at time \( t=0.1 \, [s] \) the

**Simulation Example 7.2.1**

The objective in this simulation was to test the voltage controller when it is requested to maintain a constant voltage and different loads are connected to the system.

This simulation is based on the power system shown in Figure 62 (a) and has the same parameters and compensators of the simulation example 7.2.1. So, the magnitude of the reference voltage signal is maintained at a constant value of 400 [V]. The frequency imposed for this simulation was \( f = 60 \, [Hz] \). At time \( t=0 \, [s] \) there is no load, at time \( t=0.1 \, [s] \) the
Switch 1 is closed in order to connect the Load 1, and finally at time \( t=0.2 \) \([s]\) the Switch 2 is closed to connect the Load 2.

Figure 65, shows the PCC voltage time response in the rotary frame dq, and in the stationary frame \( a-b-c \), and the real and reactive power sent to the system. The power that is sent to the system is going to change in time due to the loads added to the system, therefore from time \( t=0 \) \([s]\) to time \( t=0.1 \) \([s]\) the active and reactive power sent to the system are zero as we can see from Figure 65, afterward the system send positive real and reactive power due to the presence of Load 1, at time \( t=0.2 \) \([s]\) the real power send to the system gets higher and the reactive power becomes negative due to the presence of Load 2.
8. Voltage Control – Controlled Frequency / Unbalanced Loads

Until now we have seen that the performance of the current control and voltage control of the three-leg VSI and four-leg VSI is very similar when they are working in a balanced system.

In this chapter, the voltage control-control frequency models of the three-leg and four-leg voltage source inverter operating in autonomous mode and under unbalanced loads are studied. The same models developed in sections 5 and 7 are used for this study.

8.1 Three-Leg VSI and Four-Leg VSI

The simulations performed in this section is based on the models of Figure 42 and Figure 62 for the three-leg VSI and for the four-leg VSI respectively. The parameters considered for the RLC branch of the 3-leg VSI and 4-leg VSI are: \(L = 100[\mu H]\), \(R = 1.7[\Omega]\), and \(C = 2500[\mu F]\). The DC link voltage is \(V_{DC} = 1200[V]\). The switching frequency is \(f_s = 10[kHz]\). In the case of the 4-leg VSI, the value of inductance connected to the fourth leg is: \(L_n = 100/3[\mu H]\)

From the models presented in Figure 42 and Figure 62 we can see that there are two loads connected in parallel to the voltage capacitor through switched devices. In this simulation just the first load (inductive unbalanced load) is considered switched on. The parameters of this load is described by the table below:

<table>
<thead>
<tr>
<th>LOAD 1</th>
<th>Phase A</th>
<th>Phase B</th>
<th>Phase C</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>50 [mOhm]</td>
<td>150 [mOhm]</td>
<td>150 [mOhm]</td>
</tr>
<tr>
<td>L1</td>
<td>50 [uH]</td>
<td>80 [uH]</td>
<td>80 [uH]</td>
</tr>
</tbody>
</table>

From this table, we can see that the load in the phase A is unbalanced with respect to the other two phases.

The compensator used in the \(\alpha\)-voltage control system and \(\beta\)–voltage control system of the 3-leg and 4-leg VSI is the same compensators used in the simulations of the balanced systems performed in sections 5.2 and 7.2, which is described by the transfer function of equation (26). Likewise, the voltage compensator used in the \(\gamma\)-control system of the 4-leg VSI is the same compensator used in section 7.2 described by equation (73).

Figure 66 and Figure 67 show the time response of the PCC voltage in the rotary frame dq, and in the \(a-b-c\) stationary frame, the real and reactive power dynamic response and the current time response of the 3-leg VSI and 4-leg VSI respectively. In this simulation a constant magnitude and frequency of the PCC voltage is required under the presence of unbalanced loads as it was mentioned before.
The reference voltage employed in this simulation is $V_{ref} = 400\sin(2\pi f)$, where the reference frequency is $f = 60[Hz]$. From Figure 66 and Figure 67 we can see that the controller can track the reference voltage value under unbalanced conditions for both topologies 3-leg and 4-leg VSI.

From Figure 66 and Figure 67, we know the voltage level is maintained at the required value under the presence of unbalanced loads by the control system of each topology, but although the voltage is the same for both cases, the currents are not.

From Figure 66 we can see the currents in the 3-leg VSI are signals that can be decomposed only in the positive and negative sequences due to the power system topology. From that figure, we can appreciate that although just the load in one phase is different from the others, all three currents $i_a, i_b$, and $i_c$ are different, which means that we cannot make a phase control when we are working with the 3-leg VSI topology under unbalanced loads, instead we have to consider it as a whole system in an a two-dimensional reference frame.

Figure 67 shows the currents in the 4-leg VSI. These signals can be decomposed in positive, negative and zero sequences, which means that we are not limited to a two-dimensional reference frame. As we can appreciate from this figure, only the current $i_a$ is unbalanced, which is expected since only the load of the phase A is different. This result and the fact that currents can be reflected into a three dimensional space give us an idea that each phase voltage can be controlled independent when this configuration is employed, which is actually true.

Unlike current and voltage controllers designed in the synchronous dq-frame and presented in [4], the proportional resonant controllers designed in the stationary frame don’t require a different compensator for the positive and negative sequences when unbalanced loads are presented, computational time can be reduced since the number of compensators employed in the stationary frame control will be less than the number of compensators that d-q frame control requires. Also, the task of decomposing the signals into the positive and negative sequence is avoided when the 3-leg topology of the VSI is employed and into the positive, negative and zero sequences when the 4-leg topology of the VSI is being used.

The voltage control system made of the proportional resonant current compensator, and the proportional resonant voltage compensator are able to track perfectly the sinusoidal reference signals at a determined frequency, therefore, it doesn’t matter if the system is balanced or unbalanced, as long as the complex poles of the controller are located at the AC system frequency in order to ensure the correct operation and the tracking of the reference signal.
Figure 66. Voltage, Real/Reactive Power, and Current Dynamic Response (3-Leg VSI)
Figure 67. Voltage, Real/Reactive Power, and Current Dynamic Response (4-Leg VSI)
8.2 Four-leg VSI - Unbalanced Voltage Control / Unbalanced Loads

The advantage of using the four-leg voltage source inverter topology is that the control of the voltage and the current of each phase can be performed independently for each voltage phase.

From equation (52) we realized the average large signal model of the 4-leg VSI system is using three independent voltage control subsystems in the Alpha-Beta-Gamma frame that are not coupled each other, which is also shown in Figure 61. So, there are three degrees of freedom for the control action, one for each phase voltage.

The system model developed in PLECS for the voltage control of the 4-leg VSI in autonomous operation takes the three reference voltages in the $a$-$b$-$c$ reference frame to be transformed to the $\alpha$-$\beta$-$\gamma$ stationary frame in order to perform the control action. It is important to mention that the voltage control at the point of common coupling can be implemented also in the natural $a$-$b$-$c$ reference frame using proportional resonant compensators, but if we want to use that model we have to consider the neutral current is coupling the three subsystems in the $a$-$b$-$c$ frame in order to design the voltage controller.

This simulation (that is based on the model of Figure 62) considers the same parameters of the last section, but now the objective is controlling the voltage of each phase according to each PCC voltage phase reference signals, which are:

\[
V_{a\,\text{ref}} = 450\sin(377t) \, [V] \\
V_{b\,\text{ref}} = 400\sin(377t - \frac{2\pi}{3}) \, [V] \\
V_{c\,\text{ref}} = 350\sin(377t + \frac{2\pi}{3}) \, [V]
\]

The two loads connected in parallel to the voltage capacitor through switched devices are described by Table 9.

<table>
<thead>
<tr>
<th>LOAD 1</th>
<th>Phase A</th>
<th>Phase B</th>
<th>Phase C</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>100 [mOhm]</td>
<td>100 [mOhm]</td>
<td>50 [mOhm]</td>
</tr>
<tr>
<td>L1</td>
<td>80 [uH]</td>
<td>80 [uH]</td>
<td>50 [uH]</td>
</tr>
<tr>
<td>LOAD 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>150 [mOhm]</td>
<td>50 [mOhm]</td>
<td>150 [mOhm]</td>
</tr>
<tr>
<td>L2</td>
<td>70 [uH]</td>
<td>50 [uH]</td>
<td>70 [uH]</td>
</tr>
<tr>
<td>C2</td>
<td>10 [mF]</td>
<td>10 [mF]</td>
<td>10 [mF]</td>
</tr>
</tbody>
</table>

Figure 68 shows the reference voltage values and the voltage time response of each phase, furthermore the active and reactive power dynamic response is presented. The output currents time response send to the system is shown in Figure 69.
Figure 68. Voltage and Real/Reactive Power Time Response (Unbalances Voltages - Unbalanced Loads)
Initially the voltage control action of this simulation is performed at no–load conditions from time \( t=0 \) [s] to time \( t=0.1 \) [s]. From Figure 68 we can see that the voltage of each phase follow its own voltage reference signal with zero steady state error, also we can see that the real and reactive power send to the system is zero, as it was expected. Likewise, from Figure 69, we can see that the current send to the system is zero. At time \( t=0.1 \) [s] the first switch is closed in order to connect the Load 1 to the system. From this point, a positive values of real and reactive power are send to the system, but due to the loads are unbalanced, the real and reactive power are not constant, they present a sinusoidal behaviour as it is shown in Figure 68, where we can appreciate the voltages at PCC follow the reference with zero steady state error in this time interval. At time \( t=0.2 \) [s], the second switch is closed and the Load 2 becomes operative, the average value of the active power send to the system increases and the reactive power becomes negative due to the load 2 is dominated by the capacitor. We can see that the voltages continue following the reference signals, maintaining the required voltage in each phase despite the change in loads, showing the developed control system is working properly.
9. Conclusions

Using proportional resonant compensator in stationary frame control of grid connected voltage source inverter or VSI working in the autonomous operation mode, let us tracking sinusoidal reference signals with zero steady state error, avoiding the computational effort of synchronous d–q frame transformation. The main disadvantage of using proportional resonant compensator is the controller is tuned to just one specific frequency to perform the control action, which is not a problem for grid connected converters since the AC usually is working around the same value of frequency with just little deviations, but if the frequency of the AC system varies in a larger range the controller will not be able to track the reference value.

In the stand-alone operation of the voltage source inverter (3-leg or 4-leg), unlike the compensators designed in the dq rotary frame, stationary frame control system with proportional resonant controllers don’t require different compensators to deal with the positive and negative sequences in the case of the 3-leg VSI or positive, negative and zero in the case of the 4-leg VSI when unbalanced loads are presented. So, P+R compensator can track any periodic signal at a determined frequency.

The voltage control system developed for the stand-alone four-leg voltage source inverter let us having a precise independent control of each voltage phase at PCC no matter if the loads connected are balanced or unbalanced. Therefore the three-phase four-leg VSI can improve power quality send to the system when the converter is operating in autonomous mode. Theoretically, the four-leg voltage source inverter is able to maintain the required voltage value even in the presence of faults.

The performance of the current control of the three-leg VSI and the four-leg VSI are very similar when the reference current send to the system is balanced. So, under these conditions, the control system of the gamma axis will make the value of the gamma-current equal to zero, which is equivalent to be operating just in alpha-beta frame. Therefore the performance of the real and reactive power control system for the three-leg VSI and four-leg VSI connected to the grid is also the same when the AC system is balanced.

Since the current controller was developed in the stationary Alpha-Beta frame, the control action of real and reactive power are influenced by each other during transient states, therefore a change in the reference value of either real or active power generates a transient state in the alpha and beta components of the current that will affect both real/reactive power.

In order to avoid the mutual influence of the real/reactive power during transient states a current controller in the synchronous d-q frame could be employed, since we can make the real power proportional to the d-current, and the reactive power proportional to the q-current. This approach is also useful for voltage compensation in grid connected applications when there is the necessity to maintain a specific voltage value through sending active or reactive power according to the characteristics of the AC system.
10. References


